CHAPTER 1

Equations

Section 1A

Equations and Formulas

1-1 Variables and Expressions
Technology Lab Create a Table to Evaluate Expressions
Connecting Algebra to Geometry Perimeter
Algebra Lab Model One-Step Equations
1-2 Solving Equations by Adding or Subtracting
Connecting Algebra to Geometry Area of Composite Figures
1-3 Solving Equations by Multiplying or Dividing
Technology Lab Solve Equations by Graphing

Section 1B

Proportion and Precision

1-4 Solving Two-Step and Multi-Step Equations
Algebra Lab Model Equations with Variables on Both Sides
1-5 Solving Equations with Variables on Both Sides
Technology Lab Solve Equations Graphically
1-6 Solving for a Variable
1-7 Solving Absolute-Value Equations
1-8 Rates, Ratios, and Proportions
1-9 Applications of Proportions
1-10 Precision and Accuracy

Pacing Guide for 45-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 1

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
1-1 Lesson	Technology Lab Connecting Algebra to Geometry	Hands-On Lab 1-2 Lesson	1-2 Lesson Connecting Algebra to Geometry	1-3 Lesson
DAY 6	**DAY 7**	**DAY 8**	**DAY 9**	**DAY 10**
Technology Lab 1-4 Lesson	1-4 Lesson	Hands-On Lab 1-5 Lesson	1-5 Lesson	Technology Lab 1-6 Lesson
DAY 11	**DAY 12**	**DAY 13**	**DAY 14**	**DAY 15**
1-6 Lesson	1-7 Lesson	1-7 Lesson Multi-Step Test Prep	Ready to Go On? 1-8 Lesson	1-8 Lesson
DAY 16	**DAY 17**	**DAY 18**	**DAY 19**	**DAY 20**
1-9 Lesson	1-10 Lesson	1-10 Lesson	Multi-Step Test Prep Ready to Go On?	Chapter Review
DAY 21				
Chapter Test				

Pacing Guide for 90-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 1

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
1-1 Lesson Technology Lab Connecting Algebra to Geometry	Hands-On Lab 1-2 Lesson	Connecting Algebra to Geometry 1-3 Lesson Technology Lab	1-4 Lesson Hands-On Lab	1-5 Lesson Technology Lab
DAY 6	**DAY 7**	**DAY 8**	**DAY 9**	**DAY 10**
1-6 Lesson 1-7 Lesson	1-7 Lesson Multi-Step Test Prep	Ready to Go On? 1-8 Lesson	1-9 Lesson 1-10 Lesson	1-10 Lesson Multi-Step Test Prep Ready to Go On?
DAY 11				
Chapter Review Chapter Test				

ONGOING ASSESSMENT and INTERVENTION

	DIAGNOSE	PRESCRIBE
Assess Prior Knowledge	**Before the Chapter**	
	Diagnose readiness for the chapter.	Prescribe intervention.
	Are You Ready? SE	***Are You Ready? Intervention***
Formative Assessment	**Before Every Lesson**	
	Diagnose readiness for the lesson.	Prescribe intervention.
	Warm Up TE	**Reteach** CRB
	During Every Lesson	
	Diagnose understanding of lesson concepts.	Prescribe intervention.
	Check It Out! SE	**Reading Strategies** CRB
	Questioning Strategies TE	***Success for Every Learner***
	Think and Discuss SE	***Lesson Tutorial Videos***
	Write About It SE	
	Journal TE	
	After Every Lesson	
	Diagnose mastery of lesson concepts.	Prescribe intervention.
	Lesson Quiz TE	**Reteach** CRB
	Test Prep SE	**Test Prep Doctor** TE
	ExamView™ Assessment Suite	***Homework Help* Online**
	Before Chapter Testing	
	Diagnose mastery of concepts in chapter.	Prescribe intervention.
	Ready to Go On? SE	***Ready to Go On? Intervention***
	Multi-Step Test Prep SE	**Scaffolding Questions** TE
	Section Quizzes AR	**Reteach** CRB
	ExamView™ Assessment Suite	***Lesson Tutorial Videos***
	Before High Stakes Testing	
	Diagnose mastery of benchmark concepts.	Prescribe intervention.
	College Entrance Exam Practice SE	***College Entrance Exam Practice***
	Standardized Test Prep SE	
Summative Assessment	**After the Chapter**	
	Check mastery of chapter concepts.	Prescribe intervention.
	Multiple-Choice Tests (Forms A, B, C)	**Reteach** CRB
	Free-Response Tests (Forms A, B, C)	***Lesson Tutorial Videos***
	Performance Assessment AR	
	Cumulative Test AR	
	ExamView™ Assessment Suite	

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* **CRB** = *Chapter Resource Book* **AR** = *Assessment Resources* Available online

CHAPTER 1

Lesson Resources

Before the Lesson

Prepare

Teacher One Stop
- Editable lesson plans
- Calendar Planner
- Easy access to all chapter resources

Lesson Transparencies
- Teacher Tools

Teach the Lesson

Introduce

Alternate Openers: Explorations

Lesson Transparencies
- Warm Up
- Problem of the Day

Teach

Lesson Transparencies
- Teaching Transparencies

Know-It Notebook™
- Vocabulary
- Key Concepts

Power Presentations

Lesson Tutorial Videos SPANISH

Interactive Online Edition
- Lesson Activities
- Lesson Tutorial Videos

Lab Activities

Lab Resources Online

Online Interactivities SPANISH

TechKeys

Practice the Lesson

Practice

Chapter Resources
- Practice A, B, C

Practice and Problem Solving Workbook

IDEA Works!® Modified Worksheets and Tests

ExamView™ Assessment Suite

Homework Help Online

Online Interactivities SPANISH

Interactive Online Edition
- Homework Help

Apply

Chapter Resources
- Problem Solving

Practice and Problem Solving Workbook

Interactive Online Edition
- Chapter Project

Project Teacher Support

After the Lesson

Reteach

Chapter Resources
- Reteach
- Reading Strategies ELL

Success for Every Learner

Review

Interactive Answers and Solutions

Solutions Key

Know-It Notebook™
- Big Ideas
- Chapter Review

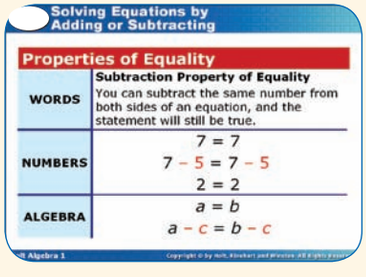
Extend

Chapter Resources
- Challenge

Technology Highlights for the Teacher

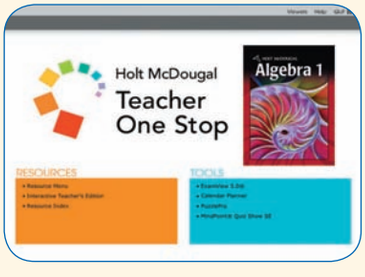
Power Presentations

Dynamic presentations to engage students. Complete PowerPoint® presentations for every lesson in this chapter.



Teacher One Stop SPANISH

Easy access to the chapter resources and assessments. Includes lesson planning software.



Interactive Online Edition

This chapter includes Tutorial Videos, Lesson Activities, Lesson Quizzes, Homework Help, and Chapter Project.

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* ELL English Language Learners SPANISH Spanish version available Available online

CHAPTER 1

Reaching All Learners

Teaching tips to help all learners appear throughout the chapter. A few that target specific students are included in the lists below.

All Learners

Lab Activities
Practice and Problem Solving Workbook
Know-It Notebook

Special Needs Students

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Inclusion ..... TE
IDEA Works!® Modified Worksheets and Tests
Ready to Go On? Intervention
Know-It Notebook
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

Developing Learners

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

On-Level Learners

Practice B ..... CRB
Problem Solving ..... CRB
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH

Advanced Learners

Practice C ..... CRB
Challenge ..... CRB
Challenge Exercises ..... SE
Reading and Writing Math *Extend* ..... TE
Are You Ready? Enrichment
Ready To Go On? Enrichment

English Language Learners

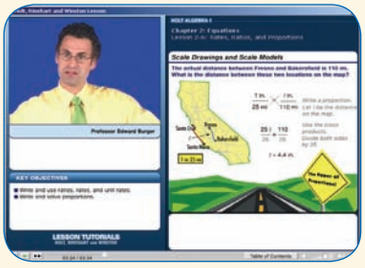
ENGLISH LANGUAGE LEARNERS

Reading Strategies ..... CRB
Are You Ready? Vocabulary ..... SE
Vocabulary Connections ..... SE
Vocabulary Review ..... SE
English Language Learners ..... TE
Success for Every Learner
Know-It Notebook
Multilingual Glossary
Lesson Tutorial Videos SPANISH

Technology Highlights for Reaching All Learners

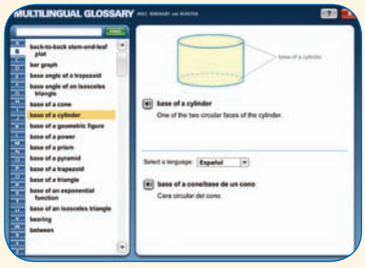
Lesson Tutorial Videos SPANISH

Starring Holt authors Ed Burger and Freddie Renfro! Live tutorials to support every lesson in this chapter.



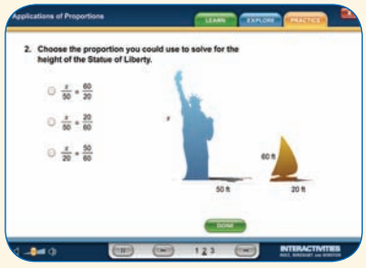
Multilingual Glossary

Searchable glossary includes definitions in English, Spanish, Vietnamese, Chinese, Hmong, Korean, and other languages.



Online Interactivities

Interactive tutorials provide visually engaging alternative opportunities to learn concepts and master skills.



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CHAPTER 1

Ongoing Assessment

Assessing Prior Knowledge

Determine whether students have the prerequisite concepts and skills for success in the chapter.

Are You Ready? ..... SE
Warm Up ..... TE

Test Preparation

Provide review and practice for the chapter and standardized tests.

Multi-Step Test Prep ..... SE
Study Guide: Review ..... SE
Test Tackler ..... SE
Standardized Test Prep ..... SE
College Entrance Exam Practice ..... SE
Countdown to Mastery ..... SE
IDEA Works!® Modified Worksheets and Tests

Alternative Assessment

Assess students' understanding of the chapter concepts and combined problem-solving skills.

Chapter Project ..... SE
Alternative Assessment ..... TE
Performance Assessment ..... AR
Portfolio Assessment ..... AR

Lesson Assessment

Provide formative assessment for each lesson of the chapter.

Questioning Strategies ..... TE
Think and Discuss ..... SE
Check It Out! Exercises ..... SE
Write About It ..... SE
Journal ..... TE
Lesson Quiz ..... TE
Alternative Assessment ..... TE
IDEA Works!® Modified Worksheets and Tests

Weekly Assessment

Provide formative assessment for each section of the chapter.

Multi-Step Test Prep ..... SE
Ready to Go On? ..... SE
Section Quizzes ..... AR
ExamView™ Assessment Suite

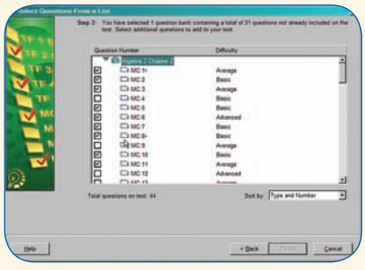
Chapter Assessment

Provide summative assessment of the chapter mastery.

Chapter Test ..... SE
Chapter Test (Levels A, B, C) ..... AR
• Multiple Choice • Free Response
Cumulative Test ..... AR
ExamView™ Assessment Suite
IDEA Works!® Modified Worksheets and Tests

Technology Highlights for Assessment

ExamView™ Assessment Suite
Use the chapter problem banks to create assessments and worksheets to print out or deliver online. Includes dynamic problems.



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CHAPTER 1

Formal Assessment

Three levels (A, B, C) of multiple-choice and free-response chapter tests, along with a performance assessment, are available in the *Assessment Resources.*

A Chapter Test

C Chapter Test

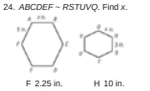
MULTIPLE CHOICE

B Chapter Test

Select the best answer.

1. A principal spent $1560 on *c* calculators for her school. Which expression shows the cost of one calculator?
   A $c + 1560$ C $1560c$
   B $1560 - c$ D $\frac{1560}{c}$
2. Which is a correct verbal expression for $8n$?
   F *n* more than 8
   G the quotient of 8 and *n*
   H the product of 8 and *n*
   J 8 increased by *n*
3. Evaluate $m + n$ for $m = 9$ and $n = 7$.
   A 2 C 63
   B 16 D 97
4. Solve $x - 74 = 146$.
   F −220 H 72
   G −72 J 220
5. Solve $19.5 = a + 2.4$.
   A −21.9 C 17.1
   B −17.1 D 21.9
6. Which equation represents the relationship "3 less than a number is −6"?
   F $n - 3 = -6$ H $3 + n = -6$
   G $3 - n = -6$ J $n - 6 = 3$
7. Solve $-11m = -132$.
   A −121 C 12
   B −12 D 121
8. Solve $-\frac{3}{7}h = 6$.
   F −14 H $2\frac{4}{7}$
   G $-2\frac{4}{7}$ J 14
9. The quotient of *n* and −4 is 8. What is the value of *n*?
   A −32 C 2
   B −2 D 32
10. Solve $6(z + 3) - 9 = 27$.
    F 0 H 5.5
    G 3 J 10.5
11. Solve $\frac{d}{14} - \frac{1}{2} = \frac{1}{14}$.
    A −8 C 6
    B −6 D 8
12. Solve $-a + 7 = 2a - 8$.
    F −3 H 5
    G $-\frac{1}{3}$ J 15
13. Tia's car needs repairs. Honest Harry will charge $70 per hour plus $130 for the part. Lucky Lou will charge $80 per hour plus $40 for the part. How long is the job if the two costs are the same?
    A $1\frac{2}{3}$ h C 17 h
    B 9 h D 20 h
14. Solve $7(x - 2) = 7x + 14$.
    F no solution H 2
    G 0 J all real numbers
15. Solve $A = \frac{1}{2}bh$ for *h*.
    A $h = \frac{A}{2b}$
    B $h = \frac{b}{2A}$
    C $h = \frac{2A}{b}$
    D $h = A - \frac{1}{2}b$

B Chapter Test *(continued)*

16. Solve $xy + 7 = n$ for *y*.
    F $y = xn - 7$
    G $y = \frac{n-7}{x}$
    H $y = x(n - 7)$
    J $y = \frac{1}{x}(n + 7)$
17. Solve $|2x - 9| = 3$.
    A 3
    B 3, 6
    C −3, −6
    D −3, 6
18. Solve $|x + 3| + 6 = 4$.
    F −1 H no solution
    G −5 J −1, −5
19. A chef can bake 15 pies in one hour. What is the rate in pies per minute?
    A 0.25 pies/min C 1.6 pies/min
    B 0.625 pies/min D 4 pies/min
20. Solve $\frac{4}{s-3} = \frac{-2}{5}$.
    F −11.5 H 8.5
    G −7 J 13
21. The ratio of girls to boys in French class is 5:4. There are 12 boys in the class. How many girls are there?
    A 7 C 15
    B 10 D 17
22. A rectangle has a length of 5 cm and a width of 3 cm. Every dimension is doubled to form a second rectangle. What is the ratio of the perimeters of the two rectangles?
    F 1:2 H 1:4
    G 1:3 I 1:8
23. Kris is 1.5 meters tall and casts a shadow 4 meters long. At the same time, a statue casts a shadow 12 meters long. What is the height of the statue?
    A 4.5 m C 24 m
    B 10.6 m D 32 m
24. *ABCDEF* ~ *RSTUVQ*. Find *x*.
    
    F 2.25 in. H 10 in.
    G 6.75 in. J 12 in.
25. Choose the more precise measurement in the pair.
    A 82 ft B 18.2 in.

Use the table for question 26.

Pane	Length (m)
1	4.112
2	4.111
3	4.110
4	4.109

26. A company that makes glass must make several very large window panes with a length of 4.110 m ± .001 m. Which pane of glass is not within the specified tolerance?
    F Pane 1 H Pane 3
    G Pane 2 J Pane 4
27. Write the possible range for the measurement 47 cm ± 2.2%.Round to the nearest hundredth if necessary.
    A 44.8 cm–49.2 cm
    B 45.97 cm–48.03 cm
    C 46.78 cm–47.22 cm

A Chapter Test

C Chapter Test

FREE RESPONSE

B Chapter Test

Solve each equation.

1. Raquel bought *p* pencils. Write an expression to show the total cost of the pencils if each pencil cost $0.75.
2. Write two verbal expressions for $b + 30$.
3. Evaluate $p - q$ for $p = 11$ and $q = 8$.
4. $x - 48 = 110$
5. $8 = a + 5.5$
6. Write an equation to represent the relationship "a number decreased by 11 is equal to −18." Then solve the equation.

Solve each equation.

7. $63 = -9m$
8. $-\frac{3}{2}t = -9$
9. Write an equation to represent the relationship "the product of a number and negative 5 is 30." Then solve the equation.

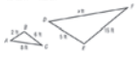
Solve each equation.

10. $32 = 12 + 4(z - 1)$
11. $\frac{3}{5} = \frac{3}{10}d - 12$
12. A landscaping company charges $100 plus $15 per hour. Another company charges $75 plus $17 per hour. How long is a job that costs the same no matter which company is used?

Solve each equation.

13. $12a - 11 = 9a - 1$
14. $5(x + 2) - 7 = 5x + 3$
15. Solve $D = \frac{M}{V}$ for *V*.
16. Solve $-8x + 4y = 28$ for *y*.
17. A boat travels 36 miles in one hour. What is its speed in miles per minute?
18. What is the solution to $|4x + 4| = 12$?
19. What is the solution to $|x - 7| + 9 = 4$?

B Chapter Test *(continued)*

20. Solve $\frac{s+1}{10} = \frac{3}{-2}$.
21. The ratio of lemon juice to water in lemonade is 1:5. If 15 cups of water are used to make a pitcher of lemonade, how many cups of lemon juice are needed?
22. A rectangle has a length of 5 cm and a width of 4 cm. Every dimension is tripled to form a second rectangle. What is the ratio of the areas of the two rectangles?
23. Nathan is 6 feet tall and casts a shadow 2.5 feet long. At the same time, a lamppost casts a shadow 10 feet long. Write and solve a proportion to find the height of the lamppost.
24. Find the value of *x* in the diagram.
    $\Delta ABC \sim \Delta DEF$
    
25. Find the value of *y* in the diagram.
    LMNP ~ QRST

Choose the more precise measurement in the pair.

26. 18 lbs; 2.39 lbs
27. 1,115 cm; 21.2 km

Use the table for questions 28 and 29.

Student	Weight (mg)
Brad	10.1
Brandon	10.23
Meagan	10

28. Three students weigh a 10.2 g copper sample in a Chemistry lab. Which student was the most precise?
29. Three students weigh a 10.2 g copper sample in a Chemistry lab. Which student was the most accurate?
30. Write the possible range of the measurement: 110 mg ± 0.5%.

A Chapter Test

C Chapter Test

PERFORMANCE ASSESSMENT

Chapter Test

Tow O' War

Purpose

This performance task assesses the student's ability to create a table to evaluate expressions.

Time

30–45 minutes

Grouping

Individuals or partners

Preparation Hints

Review numerical expressions, algebraic expressions, variables, and constants. Remind students about the use of expressions to translate words into symbols, and expressions can be used to translate real-life situations into algebraic expressions.

Overview

This task uses algebraic expressions to create a price quote chart that a tow truck company can use to give their customers a quote on how much it would cost to tow a vehicle a given number of miles. Algebraic expressions are used to figure out the amounts in a table. Students who are familiar with spreadsheets should be encouraged to practice creating the spreadsheet on their own.

Introduce the Task

Ask students if they have ever seen a table of hourly prices for renting a bicycle while on vacatioin. These tables sometimes are created by evaluating algebraic expressions. For example, if it costs $3 plus $2 per hour to rent a bicycle, than the expression $2h + 3$ can be used to find the costs for renting the bicycle for 1, 2, 3 4, ..., 24 hours.

Performance Indicators

_____ Writes expressions.
_____ Evaluate expressions.
_____ Explains how apply the expression given the context of the problem.
_____ Explains how to create a table of values using replacements for the variable.
_____ Identifies whole-number or decimal values that satisfy conditions.

Scoring Rubric

Level 4: Student solves problems correctly and gives good explanations.
Level 3: Student solves problems but does not give satisfactory explanations.
Level 2: Student solves some problems but does not give satisfactory explanations.

Chapter Test *(continued)*

Tow O' War

You are a lucky person. You saved all your money and you bought your first car. Too bad the car broke down after only one month. Because you don't have much money, you decide to shop around for the right towing company. You don't know how many miles it will be to return the car to your garage.

1. Suppose you call three companies. For each company in the tables below, fill in mileage estimates and then evaluate the given expression, where *m* represents the number of miles.

Company A	Dollar Rate $50 + m$
Number of Miles	Cost

Company B	Dollar Rate $30 + 2m$
Number of Miles	Cost

Company C	Dollar Rate $80 + 0.10m$
Number of Miles	Cost

   a. Explain why you gave those mileage estimates for each table.
   b. Explain how to evaluate the cost for each company.
2. Suppose you know how many miles your car will have to be towed, as shown in the table. Which company should you choose? Use your tables in Question 1 to choose the company that costs the least.

Mileage (mi)	Company
5	
10	
15	
20	

2. Suppose you have at most $100 to spend.
   a. Explain how you would determine the most number of miles that can be driven by each tow truck so that you keep your costs under $100.
   b. List the most number of miles in the table.

Company	Mileage
A	
B	

CHAPTER 1

MATHEMATICAL PRACTICES **Reason abstractly and quantitatively.**

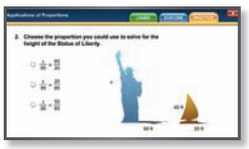
SECTION 1A
Equations and Formulas

MULTI-STEP TEST PREP Students write, solve, and graph equations to model real-world firefighting situations.

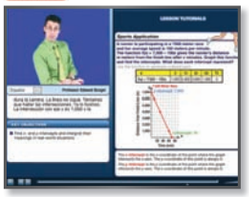
SECTION 1B
Proportion and Precision

MULTI-STEP TEST PREP Students apply ratios and proportions to model a real-world shopping situation.

Interactivities Online ▶



Video Lesson Tutorials Online



Lesson Tutorial Videos are available for EVERY example.

CHAPTER 1

Equations

1A Equations and Formulas

COMMON CORE

1-1	Variables and Expressions	A.SSE.1*
1-2	Solving Equations by Adding or Subtracting	A.REI.1
1-3	Solving Equations by Multiplying or Dividing	A.REI.1
Lab	Solve Equations by Graphing	A.REI.3
1-4	Solving Two-Step and Multi-Step Equations	A.REI.1
Lab	Model Equations with Variables on Both Sides	A.REI.1
1-5	Solving Equations with Variables on Both Sides	A.REI.1
Lab	Solve Equations Graphically	A.REI.11*
1-6	Solving for a Variable	A.CED.4*
1-7	Solving Absolute-Value Equations	A.CED.1*

1B Proportion and Precision

1-8	Rates, Ratios, and Proportions	N.Q.1*
1-9	Applications of Proportions	N.Q.1*
1-10	Precision and Accuracy	N.Q.3*

Chapter Focus

- Choose procedures to solve equations efficiently.
- Differentiate between accuracy and precision.

All in *Proportion*

A common use of equations and proportional relationships is the construction of scale models.

Learn It Online
Chapter Project Online

AP Photo/Tim Ockenden

2 *Chapter 1*

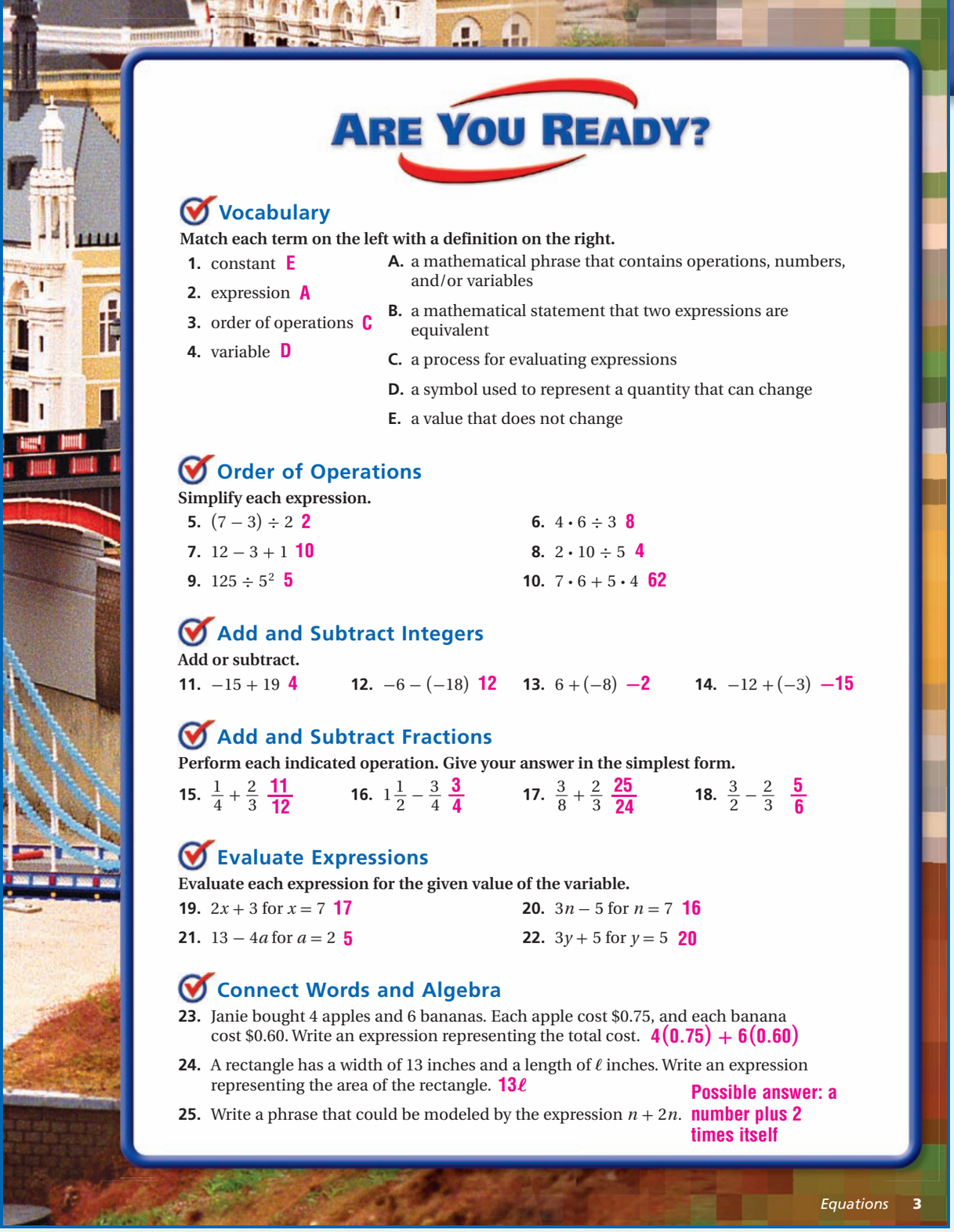
Chapter Project

A project for this chapter is available online.

Project Resources

All project resources for teachers and students are provided online.

Learn It Online
Chapter Project Online



ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. constant **E**
2. expression **A**
3. order of operations **C**
4. variable **D**

A. a mathematical phrase that contains operations, numbers, and/or variables

B. a mathematical statement that two expressions are equivalent

C. a process for evaluating expressions

D. a symbol used to represent a quantity that can change

E. a value that does not change

Order of Operations

Simplify each expression.

5. $(7 - 3) \div 2$ **2**
6. $4 \cdot 6 \div 3$ **8**
7. $12 - 3 + 1$ **10**
8. $2 \cdot 10 \div 5$ **4**
9. $125 \div 5^2$ **5**
10. $7 \cdot 6 + 5 \cdot 4$ **62**

Add and Subtract Integers

Add or subtract.

11. $-15 + 19$ **4**
12. $-6 - (-18)$ **12**
13. $6 + (-8)$ **−2**
14. $-12 + (-3)$ **−15**

Add and Subtract Fractions

Perform each indicated operation. Give your answer in the simplest form.

15. $\frac{1}{4} + \frac{2}{3}$ **$\frac{11}{12}$**
16. $1\frac{1}{2} - \frac{3}{4}$ **$\frac{3}{4}$**
17. $\frac{3}{8} + \frac{2}{3}$ **$\frac{25}{24}$**
18. $\frac{3}{2} - \frac{2}{3}$ **$\frac{5}{6}$**

Evaluate Expressions

Evaluate each expression for the given value of the variable.

19. $2x + 3$ for $x = 7$ **17**
20. $3n - 5$ for $n = 7$ **16**
21. $13 - 4a$ for $a = 2$ **5**
22. $3y + 5$ for $y = 5$ **20**

Connect Words and Algebra

23. Janie bought 4 apples and 6 bananas. Each apple cost \$0.75, and each banana cost \$0.60. Write an expression representing the total cost. **$4(0.75) + 6(0.60)$**
24. A rectangle has a width of 13 inches and a length of ℓ inches. Write an expression representing the area of the rectangle. **13ℓ**
25. Write a phrase that could be modeled by the expression $n + 2n$. **Possible answer: a number plus 2 times itself**

CHAPTER 1

ARE YOU READY?

Organizer

Objective: Assess students' understanding of prerequisite skills.

Assessing Prior Knowledge

INTERVENTION

Diagnose and Prescribe

Use this page to determine whether intervention is necessary or whether enrichment is appropriate.

Resources

- ***Are You Ready? Intervention and Enrichment* Worksheets**
- ***Are You Ready?* Online**

NO INTERVENE

ARE YOU READY?

Diagnose and Prescribe

YES ENRICH

	Are You Ready? Intervention, Chapter 1		
Prerequisite Skill	**Worksheets**	**Online**	
Order of Operations	Skill 55	Activity 55	Diagnose and Prescribe Online
Add and Subtract Integers	Skill 51	Activity 51	
Add and Subtract Fractions	Skill 48	Activity 48	
Evaluate Expressions	Skill 60	Activity 60	
Connect Words and Algebra	Skill 58	Activity 58	

***Are You Ready? Enrichment,* Chapter 1**
Worksheets
Online

Organizer

Objective: Help students organize the new concepts they will learn in this chapter.

Online Edition
Multilingual Glossary

Resources

Learn It Online
Multilingual Glossary

Possible answers to *Vocabulary Connections*

1. *Equal*, *equality*, *equalize* and *equator*; they all have something to do with being the same.
2. A literal equation might have letters in it.
3. They are the same.
4. It indicates that we must decide what we will accept as "good enough" when something is not perfect.

CHAPTER 1 Study Guide: Preview

Where You've Been

Previously, you

- practiced using operations in algebra.
- used variables to represent quantities.
- wrote expressions to represent situations.
- simplified and evaluated expressions.

In This Chapter

You will study

- how to use inverse operations to solve equations containing variables.
- writing equations to represent situations.
- simplifying equations before solving.

Where You're Going

You can use the skills in this chapter

- to compare unit prices for consumer products.
- to create or interpret scale models and drawings.
- To make measurements within an acceptable range of values.
- to solve problems in science courses and all future math courses.

Key Vocabulary/Vocabulario

accuracy	veracidad
equation	ecuación
formula	fórmula
identity	identidad
indirect measurement	medición indirecta
literal equation	ecuación literal
precision	precisión
proportion	proporción
ratio	razón
tolerance	tolerancia
unit rate	tasa unitaria

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

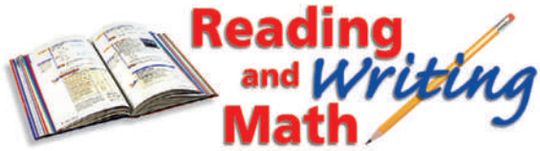
1. The word **equation** begins with the root *equa-*. List some other words that begin with *equa-*. What do all these words have in common?
2. The word *literal* means "of letters." How might a **literal equation** be different from an equation like $3 + 5 = 8$?
3. One definition of **identity** is "exact sameness." An equation consists of two expressions. If an equation is an *identity,* what do you think is true about the expressions?
4. The word *tolerate* means "to put up with" or "accept." How can this meaning help to understand the term **tolerance**?

MATHEMATICAL PRACTICES

The Common Core Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Opportunities to develop these practices are integrated throughout this program. Some examples are provided below.

1. **Make sense of problems and persevere in solving them.** Pages 9–11, 20–22, 27–30, 34, 36–38, 43–46, 51–53, 57–58, 60, 65–68, 72–74, 78–81, 82
2. **Reason abstractly and quantitatively.** Pages 9–11, 20–22, 45–46, 52, 74
3. **Construct viable arguments and critique the reasoning of others.** Pages 8, 19, 27, 28–29, 35, 38, 43, 45, 51, 52, 56, 59, 65, 67, 71, 73, 78, 80
4. **Model with mathematics.** Pages 6–8, 19, 21, 26, 34, 36, 42, 44–45, 56, 58, 72–74
5. **Use appropriate tools strategically.** Pages 16, 31, 39, 47
6. **Attend to precision.** Pages 63–64, 75–77, 78–81
7. **Look for and make use of structure.** Pages 10, 19, 26, 49, 54–55, 58
8. **Look for and express regularity in repeated reasoning.** Page 30

4 Chapter 1

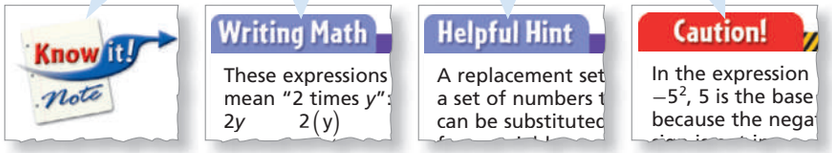


CHAPTER 1

Reading Strategy: Use Your Book for Success

Understanding how your textbook is organized will help you locate and use helpful information.

Pay attention to the **margin notes.** Know-It Note icons point out key information. Writing Math notes, Helpful Hints, and Caution notes help you understand concepts and avoid common mistakes.



The **Glossary** is found in the back of your textbook. Use it as a resource when you need the definition of an unfamiliar word or property.

The **Index** is located at the end of your textbook. Use it to locate the page where a particular concept is taught.

The **Problem-Solving Handbook** is found in the back of your textbook. These pages review strategies that can help you solve real-world problems.

Try This

Use your textbook for the following problems.

1. Use the index to find the page where a term from this chapter is defined.
2. Describe how a strategy from the Problem Solving Workbook can be used in this chapter.
3. Use the glossary to find the definition of a term from this chapter.

COMMON CORE

Standards for Mathematical Content High School

		LESSON
CREATING EQUATIONS Create equations that describe numbers or relationships		
CC.9-12.A.CED.1	Create equations and inequalities in one variable and use them to solve problems.*	1-2, 1-3, 1-4, 1-5, 1-7, 1-8, 1-9
REASONING WITH EQUATIONS AND INEQUALITIES Understand solving equations as a process of reasoning and explain the reasoning		
CC.9-12.A.REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	Lab 1-2, 1-2, 1-3, 1-4, Lab 1-5, 1-5
REASONING WITH EQUATIONS AND INEQUALITIES Solve equations and inequalities in one variable		
CC.9-12.A.REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	Lab 1-2, 1-2, 1-3, TLab 1-4, 1-4, Lab 1-5, 1-5, 1-6, 1-7, 1-8, 1-9

Organizer

Objective: Help students apply strategies to understand and retain key concepts.

INTERACTIVE **Online Edition**

ENGLISH LANGUAGE LEARNERS

Study Strategy: Use Your Own Words

Discuss Students benefit from listening to each other explain their methods for solving equations.

Encourage students to find many ways to say the same thing.

Extend As students work through this chapter, have them discuss how they would rephrase word problems in the exercises. Ask them to first divide the problem into parts, and then identify the information given and what the problem asks.

Answers to *Try This*

1–3. Check students' work.

Reading Connection

The Golden Ratio
by Mario Livio

Livio claims that phi, the Golden Ratio, is "the world's most astonishing number," and he may be right. He explains the ratio's relationships to phenomena from plant growth to galaxy structure. Helpfully, he gently debunks the belief that phi has innately aesthetic qualities.

Activity In the Fibonacci sequence, each term is the sum of the two previous terms (1, 1, 2, 3, 5, 8, 13, 21, ...). Phi is approximately 1.618034. Have students use calculators to make a list of the quotients of succeeding terms in the Fibonacci sequence (1 ÷ 1, 2 ÷ 1, 3 ÷ 2, 5 ÷ 3, etc.). They'll find that the longer their lists, the closer to phi the quotients will be.

SECTION 1A

Equations and Formulas

One-Minute Section Planner

Selected Answers in Student Edition

The answers to most odd-numbered exercises, all Check it Out! exercises, and all Chapter Study Guide: Review exercises are included in the Selected Answers section at the back of the Student Edition.

Lesson	Lab Resources	Materials
Lesson 1-1 Variables and Expressions • Translate between words and algebra. • Evaluate algebraic expressions. ☑ SAT-10 ☐ NAEP ☑ ACT ☑ SAT ☐ SAT Subject Tests		**Optional** highlighters
Technology Lab Create a Table to Evaluate Expressions • Use a table to evaluate algebraic expressions. ☑ SAT-10 ☐ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests	***Technology Lab Activities*** Lab Recording Sheet	**Required** graphing calculator, spreadsheet program
1-2 Algebra Lab Model One-Step Equations • Use algebra tiles to model and solve one-step equations. ☐ SAT-10 ☐ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests	***Algebra Lab Activities*** Lab Recording Sheet	**Required** algebra tiles (MK)
Lesson 1-2 Solving Equations by Adding or Subtracting • Solve one-step equations in one variable by using addition or subtraction. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Optional** balance scale with weights, pencil holder
Lesson 1-3 Solving Equations by Multiplying or Dividing • Solve one-step equations in one variable by using multiplication or division. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		
Technology Lab Solve Equations by Graphing • Use a graphing calculator to solve two-step equations by graphing. ☐ SAT-10 ☑ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests		**Required** graphing calculator
Lesson 1-4 Solving Two-Step and Multi-Step Equations • Solve equations in one variable that contain more than one operation. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		
Algebra Lab Model Equations with Variables on Both Sides • Use algebra tiles to model and solve equations with variables terms on both sides. ☐ SAT-10 ☐ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests	***Algebra Lab Activities*** Lab Recording Sheet	**Required** algebra tiles (MK)
Lesson 1-5 Solving Equations with Variables on Both Sides • Solve equations in one variable that contain variables on both sides. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Optional** colored pencils (MK), index cards, graphing calculator
Technology Lab Solve Equations Graphically • Solve equations in one variable that contain variables on both sides. ☐ SAT-10 ☑ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests		**Required** graphing calculator
Lesson 1-6 Solving for a Variable • Solve a formula for a given variable. • Solve an equation in two or more variables for one of the variables. ☐ SAT-10 ☑ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests		
Lesson 1-7 Solving Absolute-Value Equations • Solve equations in one variable that contain absolute-value expressions. ☐ SAT-10 ☐ NAEP ☑ ACT ☑ SAT ☐ SAT Subject Tests		

Note: If NAEP is checked, the content is tested on either the Grade 8 or the Grade 12 NAEP assessment.

MK = *Manipulatives Kit*

Math Background

COMMON CORE PROFESSIONAL DEVELOPMENT

SOLUTIONS OF EQUATIONS

Lessons 1-2 to 1-5

Before they solve equations, students need to understand what is meant by a solution. A *solution* of an equation in one variable is a value of the variable that makes the equation true. The equation $5(x + 2) - 3x = 18$, for example, is neither true nor false until a specific value replaces x. It is worthwhile to have students test various values of x in the left-hand side of the equation.

x	$5(x + 2) - 3x$
1	12
2	14
3	16
4	18

The table shows that $x = 1$, $x = 2$, and $x = 3$ are not solutions of the equation and that $x = 4$ is a solution. The one-step equations that students solve in this chapter have exactly one real-number solution, but this is not true of all equations. Lesson 1-4 introduces equations with infinitely many solutions and equations with no solutions.

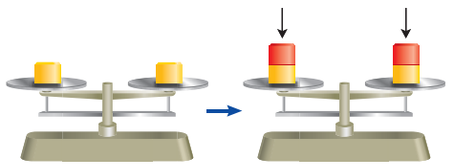
It may be useful to highlight a common mistake at this point. When asked to prove that two quantities are equal, students sometimes set the quantities equal and work to show an identity. The identity is then taken as proof that the original equation is true. This is a logical error. For example, suppose you are asked to show that $3 = 4$. Using the method described above, set $3 = 4$. Then it is true that $4 = 3$. Adding the two red equations gives $7 = 7$, which is certainly true, but that does not mean the original statement, $3 = 4$, is true. (The problem with this argument lies in the fact that not every step is reversible.)

SOLVING EQUATIONS

Lessons 1-2 to 1-5

Solving equations is one of the most important skills in K-12 mathematics. As previously shown, it is sometimes possible to find a solution by testing values of the variable, but this is generally not the most efficient solution method. In this chapter, students learn to solve an equation by transforming it into equivalent equations. *Equivalent equations* are equations that have the same solution set.

During the solution process, each transformation of an equation should be backed up by a general mathematical principle. As such, a proper justification for a step in the solution of an equation is not simply a description of what was done to the equation ("I added 3 to both sides."), but rather a general principle, such as the Addition Property of Equality ("equals added to equals are equal"), which students are introduced to in this chapter. The figure shows how this general principle may be understood in terms of a balanced scale: identical weights added to both sides of a balanced scale will preserve the balance.



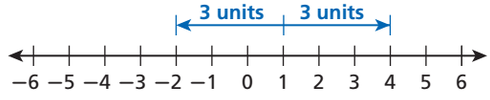
In the same way, identical quantities added to both sides of an equation will preserve the equality. A similar analogy can be made for the Subtraction Property of Equality.

ABSOLUTE-VALUE EQUATIONS

Lesson 1-7

The absolute value of a number is the number's distance from 0 on a number line. More generally, $|x - a|$ represents the distance between x and a. Thus, the equation $|x - 1| = 3$ asks, "Which values of x are 3 units away from 1?"

As the figure shows, there are two values of x that are 3 units away from 1, -2 and 4. So, this absolute-value equation has two solutions.



It is also possible for an absolute-value equation to have one solution, no solutions, or infinitely many solutions.

1-1 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Translate between words and algebra.

Evaluate algebraic expressions.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Add or subtract.

1. $6 + 104$ 110 **2.** $12 + 1.9$ 13.9

3. $23 - 8$ 15 **4.** $\frac{1}{2} + \frac{1}{5}$ $\frac{7}{10}$

Multiply or divide.

5. $324 \div 18$ 18 **6.** $\frac{1}{4}(24)$ 6

7. $13.5(10)$ 135 **8.** $18.2 \div 2$ 9.1

Also available online

Math Fact !!!

The Rhind papyrus, an ancient Egyptian scroll, was written around 1832 B.C.E. It contains algebra problems and gives conclusive evidence that algebra was used in the building of the Egyptian pyramids.

COMMON CORE

CC.9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*

CC.9-12.N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*

Learn It Online
State Resources Online

COMMON CORE

1-1 Variables and Expressions

CC.9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.* *Also* **CC.9-12.N.Q.1***

Objectives
Translate between words and algebra.

Evaluate algebraic expressions.

Vocabulary
variable
constant
numerical expression
algebraic expression
evaluate

Why learn this?
Variables and expressions can be used to determine how many plastic drink bottles must be recycled to make enough carpet for a house.

Container City, in East London, UK, is a development of buildings made from recycled shipping containers.

A home that is "green built" uses many recycled products, including carpet made from recycled plastic drink bottles. You can determine how many square feet of carpet can be made from a certain number of plastic drink bottles by using *variables*, *constants*, and *expressions*.

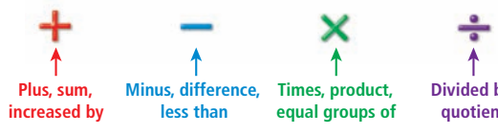
A **variable** is a letter or symbol used to represent a value that can change.

A **constant** is a value that does not change.

A **numerical expression** may contain only constants and/or operations.

An **algebraic expression** may contain variables, constants, and/or operations.

You will need to translate between algebraic expressions and words to be successful in math. The diagram below shows some of the ways to write mathematical operations with words.



EXAMPLE 1 **Translating from Algebraic Symbols to Words**

Give two ways to write each algebraic expression in words.

A $x + 3$
the sum of x and 3
x increased by 3

B $m - 7$
the difference of m and 7
7 less than m

C $2 \cdot y$
2 times y
the product of 2 and y

D $k \div 5$
k divided by 5
the quotient of k and 5

Writing Math

These expressions all mean "2 times y":
$2y$ $2(y)$
$2 \cdot y$ $(2)(y)$
$2 \times y$ $(2)y$

CHECK IT OUT! **Give two ways to write each algebraic expression in words.**

1a. $4 - n$ **1b.** $\frac{t}{5}$ **1c.** $9 + q$ **1d.** $3(h)$

Ray Roberts/Alamy Photos; (sky), PhotoDisc/Getty Images

1 Introduce

EXPLORATION

Variables and Expressions

You are going to a concert with some friends to celebrate your birthday. The tickets are $18 each.

1. Complete the table to find the total cost for different numbers of tickets.

Number of Tickets	Total Cost
1	$18
2	$36
3	$54
4	$72

2. Explain how to find the total cost for a given number of tickets.
Multiply the number of tickets by $18.

3. Suppose n represents the number of tickets. Use a combination of operations, numbers, and the letter n to represent the total cost of n tickets. 18n

4. What is the value of the mathematical statement that you wrote when $n = 6$? What does this represent?
108; the total cost of 6 tickets

THINK AND DISCUSS

5. **Explain** how to write a mathematical statement for the total cost of the tickets if each ticket costs $23.
23n, where n is the number of tickets

6. **Describe** a situation that can be represented by the mathematical statement $h + 3$.
Possible answer: The variable h represents Tim's height. Pablo is 3 cm taller than Tim, so h + 3 represents Pablo's height.

Motivate

Have students write a shopping list while you call out the items to them. Name the items quickly enough that the students have time to write down abbreviations for them, but not the entire words.

Point out to students that they probably used abbreviations or symbols to represent the words on the shopping list. Today they will learn how algebra can represent words and phrases by using variables, constants, and operation symbols.

Explorations and answers are provided in your online resources.

To translate words into algebraic expressions, look for words that indicate the action that is taking place.

Add	Subtract	Multiply	Divide
Put together, combine	Find how much more or less	Put together equal groups	Separate into equal groups

EXAMPLE 2 Translating from Words to Algebraic Symbols

A **Eve reads 25 pages per hour. Write an expression for the number of pages she reads in h hours.**

h represents the number of hours that Eve reads.

$25 \cdot h$ or $25h$ *Think: h groups of 25 pages.*

B **Sam is 2 years younger than Sue, who is y years old. Write an expression for Sam's age.**

y represents Sue's age.

$y - 2$ *Think: "younger than" means "less than."*

C **William runs a mile in 12 minutes. Write an expression for the number of miles that William runs in m minutes.**

m represents the total time William runs.

$\frac{m}{12}$ *Think: How many groups of 12 are in m?*

CHECK IT OUT!

2a. Lou drives at 65 mi/h. Write an expression for the number of miles that Lou drives in t hours. **$65t$**

2b. Miriam is 5 cm taller than her sister, who is m cm tall. Write an expression for Miriam's height in centimeters. **$m + 5$**

2c. Elaine earns \$32 per day. Write an expression for the amount that she earns in d days. **$32d$**

To **evaluate** an expression is to find its value. To evaluate an algebraic expression, substitute numbers for the variables in the expression and then simplify the expression.

EXAMPLE 3 Evaluating Algebraic Expressions

Evaluate each expression for $x = 8$, $y = 5$, and $z = 4$.

A $x + y$

$x + y = 8 + 5$ *Substitute 8 for x and 5 for y.*

$= 13$ *Simplify.*

B $\frac{x}{z}$

$\frac{x}{z} = \frac{8}{4}$ *Substitute 8 for x and 4 for z.*

$= 2$ *Simplify.*

CHECK IT OUT!

Evaluate each expression for $m = 3$, $n = 2$, and $p = 9$.

3a. mn **6** **3b.** $p - n$ **7** **3c.** $p \div m$ **3**

2 Teach

Guided Instruction

Before introducing variables and expressions, review elementary problems such as $2 + \square = 7$ and $\frac{10}{\square} = 2$. Tell students that in algebra, the box is replaced by a letter. Be sure students understand that the letter represents a number.

Teaching Tip **Visual** When students translate words into algebraic expressions, encourage them to copy the word phrases and to use a highlighter to call attention to key words.

Reaching All Learners *Through Graphic Organizers*

Copy the following table on the board and have students write real-world situations that might be modeled by each expression.

$100 - p$	$\frac{30}{x}$	$b - 1$	$\frac{m}{2}$	$6h$	$14 + s$

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Additional Examples

Example 1

Give two ways to write each algebraic expression in words.

Possible answers given.

A. $9 + r$ the sum of 9 and r; 9 increased by r

B. $q - 3$ the difference of q and 3; 3 less than q

C. $7m$ the product of m and 7; m times 7

D. $j \div 6$ the quotient of j and 6; j divided by 6

Example 2

A. John types 62 words per minute. Write an expression for the number of words he types in m minutes. $62m$

B. Roberto is 4 years older than Emily, who is y years old. Write an expression for Roberto's age. $y + 4$

C. Joey earns \$5 for each car he washes. Write an expression for the number of cars Joey must wash to earn d dollars. $\frac{d}{5}$

Example 3

Evaluate each expression for $a = 4$, $b = 7$, and $c = 2$.

A. $b - c$ 5 **B.** ac 8

Also available online

Mathematical Practices **Questioning Strategies**

EXAMPLE 1

- What words are indicated by each operation symbol?

EXAMPLE 2

- What clues tell you which operation to use?

EXAMPLE 3

- Explain why the value of an algebraic expression is numeric.

Answers to *Check It Out!*

Possible answers given.

1a. 4 decreased by n; n less than 4

1b. the quotient of t and 5; t divided by 5

1c. the sum of 9 and q; q added to 9

1d. the product of 3 and h; 3 times h

Power Presentations with PowerPoint®

Additional Examples

Example 4

Approximately eighty-five 20-ounce plastic bottles must be recycled to produce the fiberfill for a sleeping bag.

A. Write an expression for the number of bottles needed to make s sleeping bags.
$85s$

B. Find the number of bottles needed to make 20, 50, and 325 sleeping bags.
1700; 4250; 27,625

Also available online

Questioning Strategies

EXAMPLE 4

- How do you think the letter for the variable was chosen?
- Why is multiplication used in the expression?

EXAMPLE 4 Recycling Application

Approximately fourteen 20-ounce plastic drink bottles must be recycled to produce 1 square foot of carpet.

a. Write an expression for the number of bottles needed to make c square feet of carpet.

The expression $14c$ models the number of bottles needed to make c square feet of carpet.

b. Find the number of bottles needed to make 40, 120, and 224 square feet of carpet.

Evaluate $14c$ for $c = 40$, 120, and 224.

c	$14c$
40	$14(40) = 560$
120	$14(120) = 1680$
224	$14(224) = 3136$

To make 40 ft^2 of carpet, 560 bottles are needed.
To make 120 ft^2 of carpet, 1680 bottles are needed.
To make 224 ft^2 of carpet, 3136 bottles are needed.

Helpful Hint

A *replacement set* is a set of numbers that can be substituted for a variable. The replacement set in Example 4 is {40, 120, 224}.

CHECK IT OUT!

4. To make one sweater, sixty-three 20-ounce plastic drink bottles must be recycled.

a. Write an expression for the number of bottles needed to make s sweaters. **$63s$**

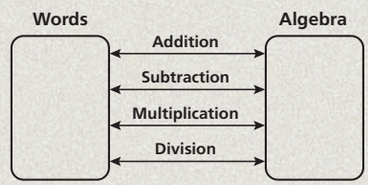
b. Find the number of bottles needed to make 12, 25, and 50 sweaters. **756 bottles; 1575 bottles; 3150 bottles**

THINK AND DISCUSS

1. Write two ways to suggest each of the following, using words or phrases: addition, subtraction, multiplication, division.

2. Explain the difference between a numerical expression and an algebraic expression.

3. GET ORGANIZED Copy and complete the graphic organizer. Next to each operation, write a word phrase in the left box and its corresponding algebraic expression in the right box.



(tc), Sam Dudgeon/HMH; (tr), Thinkstock/Getty

3 Close

Summarize

Have students list key words for each operation. Possible answers: addition—*add*, *sum*; subtraction—*minus*, *difference*; multiplication—*times*, *product*; division—*divided by*, *quotient* Have students explain the difference between a variable and a constant. A variable is a value that can change. A constant is a value that does not change.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

Possible answers:

1. addition—*increased by, sum of*; subtraction—*decreased by, difference of*; multiplication—*multiplied by, product of*; division—*divided by, quotient of*
2. Both types of expressions may contain numbers and operations. Algebraic expressions may also contain variables.
3. See Additional Answers.

1-1 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** A(n) ___?___ is a value that can change. (*algebraic expression*, *constant*, or *variable*) variable

SEE EXAMPLE 1

Give two ways to write each algebraic expression in words.

2. $n - 5$ 5 less than n; n decreased by 5
3. $\frac{f}{3}$ the quotient of f and 3; f divided by 3
4. $c + 15$ c increased by 15; the sum of c and 15
5. $9 - y$ 9 decreased by y; y less than 9
6. $\frac{x}{12}$
7. $t + 12$
8. $8x$
9. $x - 3$

SEE EXAMPLE 2

10. George drives at 45 mi/h. Write an expression for the number of miles George travels in h hours. $45h$
11. The length of a rectangle is 4 units greater than its width w. Write an expression for the length of the rectangle. $w + 4$

SEE EXAMPLE 3

Evaluate each expression for $a = 3$, $b = 4$, and $c = 2$.

12. $a - c$ 1
13. ab 12
14. $b \div c$ 2
15. ac 6

SEE EXAMPLE 4

16. Brianna practices the piano 30 minutes each day.
    a. Write an expression for the number of hours she practices in d days. $0.5d$
    b. Find the number of hours Brianna practices in 2, 4, and 10 days. 1; 2; 5

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–24	1
25–26	2
27–30	3
31	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Give two ways to write each algebraic expression in words.

17. $5p$
18. $4 - y$
19. $3 + x$
20. $3y$
21. $-3s$
22. $r \div 5$
23. $14 - t$
24. $x + 0.5$
25. Friday's temperature was 20° warmer than Monday's temperature t. Write an expression for Friday's temperature. $t + 20$
26. Ann sleeps 8 hours per night. Write an expression for the number of hours Ann sleeps in n nights. $8n$

Evaluate each expression for $r = 6$, $s = 5$, and $t = 3$.

27. $r - s$ 1
28. $s + t$ 8
29. $r \div t$ 2
30. sr 30
31. Jim is paid for overtime when he works more than 40 hours per week.
    a. Write an expression for the number of hours he works overtime when he works h hours. $h - 40$
    b. Find the number of hours Jim works overtime when he works 40, 44, 48, and 52 hours. 0; 4; 8; 12
32. **Write About It** Write a paragraph that explains to another student how to evaluate an expression. To evaluate an expression is to find its value. To do this, substitute values for the variables and perform all the indicated operations.

Write an algebraic expression for each verbal expression. Then write a real-world situation that could be modeled by the expression.

33. the product of 2 and x
34. b less than 17
35. 10 more than y

Teacher to Teacher

I like to use the Think and Discuss questions from the previous lesson as a warm-up activity to begin the next class. It helps students refresh their memory of the concepts just learned and practiced. The Think and Discuss questions often begin very useful class discussions and allow students to learn from their peers.

John Bakelaar
Jackson, MS

1-1 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–2
Basic 17–26, 41–43
Average 17–26, 40–43
Advanced 17–26, 40–43

If you finished Examples 1–4
Basic 17–31, 33, 34, 36–38, 40–43, 48–50
Average 17–35, 36–40 even, 44, 48–51
Advanced 18–26 even, 27–36, 39, 40, 44–54

Homework Quick Check
Quickly check key concepts.
Exercises: 18, 24, 26, 28, 31, 40

Answers

6–9. Possible answers given.
6. one-twelfth x; the quotient of x and 12
7. the sum of t and 12; t increased by 12
8. the product of 8 and x; 8 groups of x
9. x decreased by 3; the difference of x and 3

17–24, 33–35. See Additional Answers.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 10–11, 16, 25–26, 31, 36–38, 42–43, 54

Construct viable arguments and critique the reasoning of others. Exercises 32, 44

Look for and make use of structure. Exercise 38

Learn It Online
State Resources Online

MULTI-STEP TEST PREP **Exercise 36** involves using algebraic expressions to describe the pressure on a diver's body at different depths. The exercise prepares students for the Multi-Step Test Prep.

Teaching Tip **Geometry** In **Exercise 37,** students must write an expression for the area of a rectangle. Remind students that the area of a rectangle is its length times its width.

MULTI-STEP TEST PREP

36. The air around you puts pressure on your body equal to 14.7 pounds per square inch (psi). When you are underwater, the water exerts additional pressure on your body. For each foot you are below the surface of the water, the pressure increases by 0.445 psi.

a. What does 14.7 represent in the expression $14.7 + 0.445d$? **air pressure**

b. What does d represent in the expression? **depth below the water in feet**

c. What is the total pressure exerted on a person's body when $d = 8$ ft? **18.26 psi**

37. Geometry The length of a rectangle is 9 inches. Write an expression for the area of the rectangle if the width is w inches. Find the area of the rectangle when the width is 1, 8, 9, and 11 inches. **$9w$; 9 in^2; 72 in^2; 81 in^2; 99 in^2**

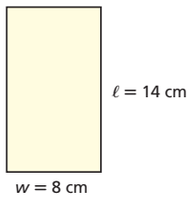
38. Geometry The perimeter of any rectangle is the sum of its lengths and widths. The area of any rectangle is the length ℓ times the width w.

a. Write an expression for the perimeter of a rectangle. **$2\ell + 2w$**

b. Find the perimeter of the rectangle shown. **44 cm**

c. Write an expression for the area of a rectangle. **ℓw**

d. Find the area of the rectangle shown. **112 cm^2**



Complete each table. Evaluate the expression for each value of x.

39.

x	$x + 12$
1	13
2	14
3	15
4	16

40.

x	$10x$
1	10
5	50
10	100
15	150

41.

x	$x \div 2$
12	6
20	10
26	13
30	15

LINK Astronomy

A crater on Canada's Devon Island is geologically similar to the surface of Mars. However, the temperature on Devon Island is about 37 °F in summer, and the average summer temperature on Mars is −85 °F.

42. Astronomy An object's weight on Mars can be found by multiplying 0.38 by the object's weight on Earth. **$0.38p$**

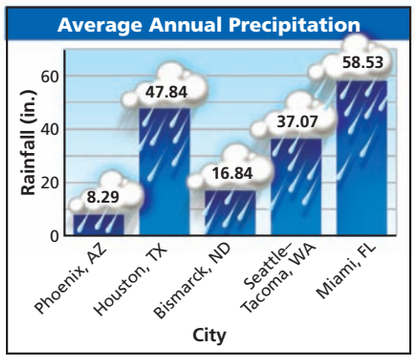
a. An object weighs p pounds on Earth. Write an expression for its weight on Mars.

b. Dana weighs 120 pounds, and her bicycle weighs 44 pounds. How much would Dana and her bicycle together weigh on Mars? **62.32 lb**

43. Meteorology Use the bar graph to write an expression for the average annual precipitation in New York, New York.

a. The average annual precipitation in New York is m inches more than the average annual precipitation in Houston, Texas. **$47.84 + m$**

b. The average annual precipitation in New York is s inches less than the average annual precipitation in Miami, Florida. **$58.53 - s$**



(bl), AP Photo/NASA Haughton-Mars Project 2001, Pascal Lee; (tl), Royalty Free/CORBIS

PRACTICE A

PRACTICE C

PRACTICE B

Give two ways to write each algebraic expression in words.

1. $15 - b$
   the difference of 15 and b
   b less than 15

2. $\frac{x}{16}$
   the quotient of x and 16
   x divided by 16

3. $x + 9$
   the sum of x and 9
   9 more than x

4. $(2)(t)$
   the product of 2 and t
   2 times t

5. $z - 7$
   the difference of z and 7
   7 less than z

6. $4y$
   the product of 4 and y
   4 times y

7. Sophie's math class has 6 fewer boys than girls, and there are g girls. Write an expression for the number of boys. **$g - 6$**

8. A computer printer can print 10 pages per minute. Write an expression for the number of pages the printer can print in m minutes. **$10m$**

Evaluate each expression for $r = 8$, $s = 2$, and $t = 5$.

9. st **10**
10. $r - s$ **4**
11. $s + t$ **7**
12. $r - t$ **3**
13. $r \cdot s$ **16**
14. $t - s$ **3**

15. Paula always withdraws 20 dollars more than she needs from the bank.
    a. Write an expression for the amount of money Paula withdraws if she needs d dollars. **$d + 20$**
    b. Find the amount of money Paula withdraws if she needs 20, 60, and 75 dollars. **40 dollars; 80 dollars; 95 dollars**

READING STRATEGIES

To translate between phrases and algebraic expressions, you must connect words with symbols. Look at the examples in the table below.

Operations	Words	Symbols
Addition	"sum of 3 and n" "n more than 3" "3 plus n"	$3 + n$
Subtraction	"difference of 3 and n" "n less than 3" "3 minus n"	$3 - n$
Multiplication	"product of 3 and n" "3 times n" "3 groups of n"	$3n$ $3 \cdot n$ $3 \times n$ $3(n)$
Division	"quotient of 3 and n" "3 divided by n" "3 separated into n groups"	$3 \div n$ $\frac{3}{n}$

Solve each problem.

1. Write "10 times the value of y" in symbols in three different ways.
   Possible answers: $10y$, $10 \cdot y$, and $10(y)$

2. Write "$k \div 6$" in words in two different ways.
   Possible answers: k divided by 6, the quotient of k and 6

3. Write "4 less than b" in symbols. **$b - 4$**

4. Write "b less than 4" in symbols. **$4 - b$**

Translate each word phrase into an algebraic expression.

5. Craig types 20 words per minute. Write an expression for the number of words Craig types in m minutes. **$20m$**

6. Jeana is 58 inches tall. Her sister Janelle is t inches taller. Write an expression for Janelle's height. **$58 + t$**

RETEACH

To translate words into algebraic expressions, find words like these that tell you the operation.

+	−	·	÷
add	subtract	multiply	divide
sum	difference	product	quotient
more	less	times	split
increased	decreased	per	ratio

Kenny owns v video games. Stan owns 7 more video games than Kenny. Write an expression for the number of video games Stan owns.

v represents the number of video games Kenny owns.

$v + 7$ *Think: The word "more" indicates addition.*

Order does not matter for addition. The expression $7 + v$ is also correct.

Jenny is 12 years younger than Candy. Write an expression for Jenny's age if Candy is c years old.

c represents Candy's age.

The word "younger" means "less," which indicates subtraction.

$c - 12$ *Think: Candy is older, so subtract 12 from her age.*

Order does matter for subtraction. The expression $12 - c$ is incorrect.

1. Jared can type 35 words per minute. Write an expression for the number of words he can type in m minutes. **$35m$**

2. Mr. O'Brien's commute to work is 0.5 hour less than Miss Santos's commute. Write an expression for the length of Mr. O'Brien's commute if Miss Santos's commute is h hours. **$h - 0.5$**

3. Mrs. Knighten bought a box of c cookies and split them evenly between the 25 students in her classroom. Write an expression for the number of cookies each student received. **$c \div 25$**

4. Enrique collected 152 recyclable bottles, and Latasha collected b recyclable bottles. Write an expression for the number of bottles they collected altogether. **$152 + b$**

5. Tammy's current rent is r dollars. Next month it will be reduced by $50. Write an expression for next month's rent in dollars. **$r - 50$**

44. Critical Thinking Compare algebraic expressions and numerical expressions. Give examples of each. **Both algebraic and numerical expressions contain numbers and operations, but algebraic expressions also contain variables.**

Write an algebraic expression for each verbal expression. Then evaluate the algebraic expression for the given values of *x*.

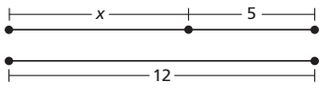
	Verbal	Algebraic	$x = 12$	$x = 14$
	x reduced by 5	$x - 5$	$12 - 5 = 7$	$14 - 5 = 9$
45.	7 more than x	$x + 7$	19	21
46.	The quotient of x and 2	$\frac{x}{2}$	6	7
47.	The sum of x and 3	$x + 3$	15	17

TEST PREP

48. Claire has had her driver's license for 3 years. Bill has had his license for b fewer years than Claire. Which expression can be used to show the number of years Bill has had his driver's license?

Ⓐ $3 + b$ Ⓑ $b + 3$ Ⓒ $3 - b$ Ⓓ $b - 3$

49. Which expression represents x?



Ⓕ $12 - 5$ Ⓗ $12(5)$

Ⓖ $12 + 5$ Ⓙ $12 \div 5$

50. Which situation is best modeled by the expression $25 - x$?

Ⓐ George places x more video games on a shelf with 25 games.

Ⓑ Sarah has driven x miles of a 25-mile trip.

Ⓒ Amelia paid 25 dollars of an x dollar lunch that she shared with Ariel.

Ⓓ Jorge has 25 boxes full of x baseball cards each.

CHALLENGE AND EXTEND

Evaluate each expression for the given values of the variables.

51. $2ab$; $a = 6$, $b = 3$ **36** **52.** $2x + y$; $x = 4$, $y = 5$ **13** **53.** $3x \div 6y$; $x = 6$, $y = 3$ **1**

54. Multi-Step An Internet service provider charges \$9.95/month for the first 20 hours and \$0.50 for each additional hour. Write an expression representing the charges for h hours of use in one month when h is more than 20 hours. What is the charge for 35 hours? **$9.95 + 0.50(h - 20)$; \$17.45**

TEST PREP DOCTOR For **Exercise 50,** students may recognize choices **B** and **C** as indicating subtraction. Show students that since 25 is written first in the expression, 25 is the full amount being considered. In **C,** \$25 is only part of the cost of lunch, but in **B,** 25 miles is the whole trip, so **B** is the answer.

Journal

Have students describe some things in their lives that change, and then relate them to the algebraic definition of "variable."

ALTERNATIVE ASSESSMENT

Instruct students to look through newspapers and magazines to find real-world situations that can be described using words that indicate mathematical operations. Then have them write how they would model each situation with an algebraic expression.

Power Presentations with PowerPoint®

1-1 Lesson Quiz

Give two ways to write each algebraic expression in words.

1, 2. Possible answers given.

1. $j - 3$ the difference of j and 3; 3 less than j

2. $4p$ 4 times p; the product of 4 and p

3. Mark is 5 years older than Juan, who is y years old. Write an expression for Mark's age. $y + 5$

Evaluate each expression for $c = 6$, $d = 5$, and $e = 10$.

4. $\frac{d}{e}$ $\frac{1}{2}$ **5.** $c + d$ 11

Shemika practices basketball for 2 hours each day.

6. Write an expression for the number of hours she practices in d days. $2d$

7. Find the number of hours she practices in 5, 12, and 20 days. 10 hours; 24 hours; 40 hours

Also available online

PROBLEM SOLVING

Write the correct answer.

1. For her book club, Sharon reads for 45 minutes each day. Write an expression for the number of hours she reads in d days.
   $45d$
2. The minimum wage in 2003 was \$5.15. This was w more than the minimum wage in 1996. Write an expression for the minimum wage in 1996.
   $5.15 - w$
3. According to the 2000 census, the number of people per square mile in Florida was about 216 more than the number of people per square mile in Texas. Write an expression for the number of people per square mile in Florida if there were t people per square mile in Texas.
   $216 + t$
4. The cost of a party is \$550. The price per person depends on how many people attend the party. Write an expression for the price per person if p people attend the party. Then find the price per person if 25, 50, and 55 people attend the party.
   $\frac{550}{p}$
   \$22, \$11, \$10

Use the table below to answer questions 5–6, which shows the years five states entered the Union. Select the best answer.

State	Year Entered into Union
Florida	1845
Indiana	1816
Pennsylvania	1787
Texas	1845
West Virginia	1863

5. North Carolina entered the Union x years after Pennsylvania. Which expression shows the year North Carolina entered the Union?
   A $1845 + x$ **C** $1787 + x$
   B $1845 - x$ D $1787 - x$
6. The expression $f - 26$ represents the year Alabama entered the Union, where f is the year Florida entered. In which year did Alabama enter the Union?
   F 1819 H 1837
   G 1826 J 1871
7. The number of states that entered the Union in 1889 was half the number of states s that entered in 1788. Which expression shows the number of states that entered the Union in 1889?
   A $2s$ C $s + 2$
   B $s \div 2$ D $2 - s$

CHALLENGE

A cell phone company is offering two different monthly plans. Each plan charges a monthly fee and then an additional cost per minute.

Plan A	\$40 fee	45 cents per minute
Plan B	\$70 fee	35 cents per minute

The mathematical expression for Plan A that represents the charges for the monthly fee plus the charge per minute m for one month is:

$40 + 0.45(m)$

1. Write the expression which represents the charges for the monthly fee plus the charge per minute m for Plan B for one month.
   $70 + 0.35(m)$

Use the expressions above to complete the chart.

Number of Minutes	Expression for Plan A	Cost of Plan A	Expression for Plan B	Cost for Plan B
100	$40 + 0.45(100)$	\$85	$70 + 0.35(100)$	\$105
200	$40 + 0.45(200)$	\$130	$70 + 0.35(200)$	\$140
300	$40 + 0.45(300)$	\$175	$70 + 0.35(300)$	\$175
400	$40 + 0.45(400)$	\$220	$70 + 0.35(400)$	\$210
500	$40 + 0.45(500)$	\$265	$70 + 0.35(500)$	\$245

2. Which plan is less expensive for 200 minutes or less each month?
   Plan A
3. Which plan is less expensive for 400 minutes or more each month?
   Plan B
4. Suppose a friend asks you which of the two plans is less expensive. How would you respond?
   Possible answer: It depends on the number of minutes used. Plan A is cheaper if you use less than 300 minutes each month. Plan B is cheaper if you use more than 300 minutes each month. If you use exactly 300 minutes each month, the plans cost the same.

Technology LAB Organizer

Use with the lesson *Variables and Expressions*

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use a table to evaluate algebraic expressions.

Materials: graphing calculator, spreadsheet program

Online Edition
TechKeys

Countdown

Resources

Technology Lab Activities
Lab Recording Sheet

Teach

Discuss

Viewing a table of values can help students see a pattern. When using technology to create a table, use an asterisk (*) for multiplication, a forward slash (/) for division, and a caret (^) for exponentiation.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

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State Resources Online

1-1 Technology LAB

Create a Table to Evaluate Expressions

You can use a graphing calculator to quickly evaluate expressions for many values of the variable.

Use with Variables and Expressions

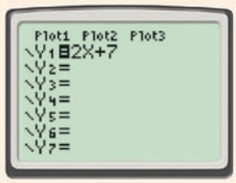
Learn It Online
Lab Resources Online

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

Activity 1

Evaluate $2x + 7$ for $x = 25, 125, 225, 325,$ and 425.

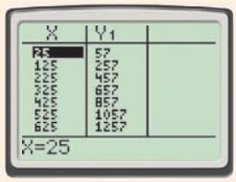
1. Press **Y=** and enter **2X+7** for **Y1**.



2. Determine a pattern for the values of x.
The x-values start with 25 and increase by 100.

3. Press **2nd** **WINDOW** (TBLSET) to view the *Table Setup* window.
Enter **25** as the starting value in **TblStart=**.
Enter **100** as the amount by which x changes in **△Tbl=**.

4. Press **2nd** **GRAPH** (TABLE) to create a table of values.
The first column shows values of x starting with 25 and increasing by 100.
The second column shows values of the expression $2x + 7$ when x is equal to the value in the first column.
You can use the arrow keys to view the table when x is greater than 625.



Try This

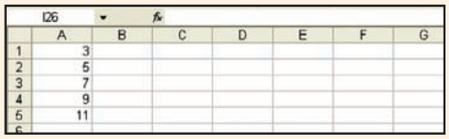
1. Use the table feature of a graphing calculator to evaluate $5x - 7$ for $x = 4, 6, 8, 10,$ and 12. **13; 23; 33; 43; 53**
   a. What value did you enter in **TblStart=?** **4**
   b. What value did you enter in **△Tbl=?** **2**
2. Use the table feature of a graphing calculator to evaluate $3x + 4$ for $x = -5, -1, 3, 7,$ and 11. **−11; 1; 13; 25; 37**
   a. What value did you enter in **TblStart=?** **−5**
   b. What value did you enter in **△Tbl=?** **4**

You can also use a spreadsheet program to evaluate expressions.

Activity 2

Evaluate $2x + 7$ for $x = 3, 5, 7, 9,$ and 11.

1. In the first column, enter the values 3, 5, 7, 9, and 11.

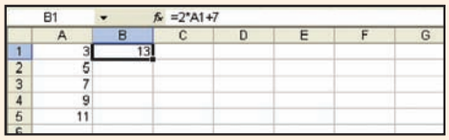


2. Enter the expression in cell B1.

   To do this, type the following:
   = 2 * A1 + 7

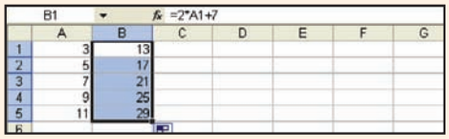
3. Press Enter.

   The value of $2x + 7$ when $x = 3$ appears in cell B1.

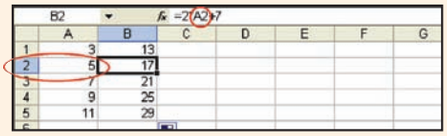


4. Copy the formula into cells B2, B3, B4, and B5.

   Use the mouse to click on the lower right corner of cell B1. Hold down the mouse button and drag the cursor through cell B5.



   For each row in column B, the number that is substituted for x is the value in the same row of column A.



   You can continue the table by entering more values in column A and copying the formula from B1 into more cells in column B.

Try This

3. Use a spreadsheet program to evaluate $-2x + 9$ for $x = -5, -2, 1, 4,$ and 7. **19; 13; 7; 1; –5**
   a. What values did you enter in column A? **–5; –2; 1; 4; 7**
   b. What did you type in cell B1? **= –2*A1 + 9**
4. Use a spreadsheet program to evaluate $7x - 10$ for $x = 2, 7, 12, 17,$ and 22. **4; 39; 74; 109; 144**
   a. What values did you enter in column A? **2; 7; 12; 17; 22**
   b. What did you type in cell B1? **= 7*A1 – 10**

Close

Key Concept

The table feature of a graphing calculator or a spreadsheet can be used to evaluate an expression for many values of the variable(s) at one time.

Assessment

Journal Have students explain how to use a graphing calculator and a spreadsheet program to evaluate expressions.

Organizer

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Apply algebra skills of simplifying expressions to finding perimeters of polygons.

Online Edition

Teach

Remember

Students review and apply the concept of perimeter as the sum of the side lengths of a polygon.

Teaching Tip **Inclusion** To make sure every side of the polygon is added, have students write a blank that can be filled in with an expression for the length of each side. For example, a triangle has three sides, so students should first write three blanks:

_____ + _____ + _____

Close

Assess

Have students draw a rectangle and label the length and width with algebraic expressions. Have them write and simplify an expression for the perimeter. Then have them double the length of each side and write and simplify an expression for the perimeter of the new figure.

Learn It Online
State Resources Online

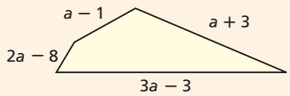
Perimeter

The distance around a geometric figure is called the *perimeter*. You can use what you have learned about combining like terms to simplify expressions for perimeter.

A closed figure with straight sides is called a *polygon*. To find the perimeter of a polygon, add the lengths of the sides.

Example 1

A **Write an expression for the perimeter of the quadrilateral.**



Add the lengths of the four sides.

$P = (a + 3) + (2a - 8) + (3a - 3) + (a - 1)$

Combine like terms to simplify.

$P = (a + 2a + 3a + a) + (3 - 8 - 3 - 1)$

$= 7a - 9$ *This is a general expression for the perimeter.*

B **Find the perimeter of this quadrilateral for $a = 5$.**

Substitute 5 for a.

$P = 7(5) - 9$ *Multiply; then subtract.*

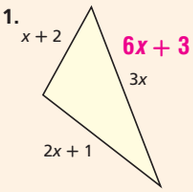
$= 35 - 9$

$= 26$ *This is the perimeter when a = 5.*

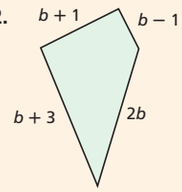
Try This

Write and simplify an expression for the perimeter of each figure.

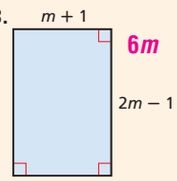
1. $6x + 3$



2. $5b + 3$

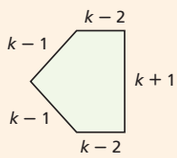


3. $6m$

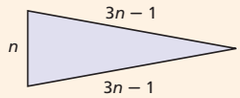


Find the perimeter of each figure for the given value of the variable.

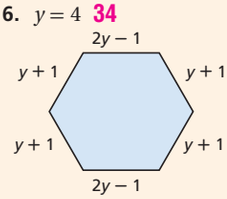
4. $k = 3$ **10**



5. $n = 10$ **68**



6. $y = 4$ **34**

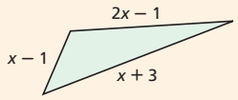


Combining like terms is one way to explore what happens to the perimeter when you double the sides of a triangle or other polygon.

Example 2

What happens to the perimeter of this triangle when you double the length of each side?

Write an expression for the perimeter of the smaller triangle.
Combine like terms to simplify the expression.

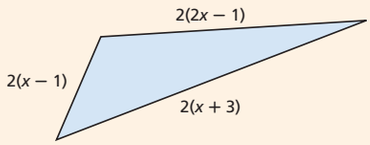


$(x-1)+(2x-1)+(x+3)$

$(x+2x+x)+(-1-1+3)$

$4x+1$ *Perimeter of small triangle*

Double the length of each side of the triangle.



$2(x-1)=2x-2$

$2(2x-1)=4x-2$

$2(x+3)=2x+6$

Find the perimeter of the larger triangle.
Combine like terms to simplify.

$(2x-2)+(4x-2)+(2x+6)$ *Add the lengths of the sides.*

$(2x+4x+2x)+(-2-2+6)$ *Use the Associative and Commutative Properties of Addition and combine like terms.*

$8x+2$ *Perimeter of large triangle*

Use the Distributive Property to show that the new perimeter is twice the original perimeter.

$8x+2=2(4x+1)$

Try This

Each set of expressions represents the side lengths of a triangle. Use the Distributive Property to show that doubling the side lengths doubles the perimeter.

7. $2p+1$, $3p+2$, $5p$

8. $c-1$, $2c+1$, $3c-1$

9. $w+5$, $w+5$, $3w-1$

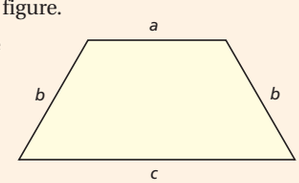
10. $h-2$, $3h$, $2h+3$

Solve each problem.

11. Use the triangles in Example 2. Find the side lengths and perimeters for $x=5$.

12. The sides of a quadrilateral are $2x-1$, $x+3$, $3x+1$, and $x-1$. Double the length of each side. Then find an expression for the perimeter of the new figure.
$4x-2$, $2x+6$, $6x+2$, $2x-2$; $14x+4$

13. What happens to the perimeter of this trapezoid when you triple the length of each side? Use the variables *a*, *b*, *b*, and *c* for the lengths of the sides. Explain your answer using the Distributive Property.
It triples; $a+b+b+c=a+2b+c$; $3a+3b+3b+3c=3a+6b+3c$; $3a+6b+3c=3(a+2b+c)$



Answers

7. Perimeter: $10p+3$; Double perimeter: $20p+6$; Distrib. Prop: $20p+6=2(10p+3)$

8. Perimeter: $6c-1$; Double perimeter: $12c-2$; Distrib. Prop: $12c-2=2(6c-1)$

9. Perimeter: $5w+9$; Double perimeter: $10w+18$; Distrib. Prop: $10w+18=2(5w+9)$

10. Perimeter: $6h+1$; Double perimeter: $12h+2$; Distrib. Prop: $12h+2=2(6h+1)$

11. small triangle: 9, 4, 8; 21; large triangle: 18, 8, 16; 42

Organizer

Use with the lesson Solving Equations by Adding or Subtracting

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use algebra tiles to model and solve one-step equations.

Materials: algebra tiles

Online Edition
Algebra Tiles

Countdown

Resources

Algebra Lab Activities
Lab Recording Sheet

Teach

Discuss

Encourage students to first state which tile they are going to isolate. Algebra tiles can be found in the Manipulatives Kit (MK).

Alternative Approach

Use the transparency mat and transparency algebra tiles (MK).

Close

Key Concept

You can solve one-step equations by first adding the same number of tiles to both sides to create zero pairs.

Assessment

Journal Have students explain how to model and solve one-step equations with algebra tiles.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. *Also* **CC.9-12.A.REI.3**

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State Resources Online

1-2

Model One-Step Equations

You can use algebra tiles and an equation mat to model and solve equations. To find the value of the variable, place or remove tiles to get the x-tile by itself on one side of the mat. You must place or remove the same number of yellow tiles or the same number of red tiles on both sides.

Use with Solving Equations by Adding or Subtracting

KEY

+ = 1
− = −1
+ = x

REMEMBER

+ + − = 0

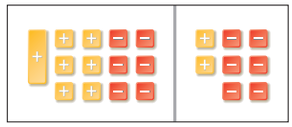
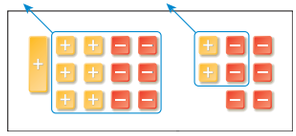
Learn It Online
Lab Resources Online

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. *Also* **CC.9-12.A.REI.3**

Activity

Use algebra tiles to model and solve $x + 6 = 2$.

MODEL	ALGEBRA
Model $x + 6$ on the left side of the mat and 2 on the right side of the mat.	$x + 6 = 2$
 *Place 6 red tiles on both sides of the mat. This represents adding -6 to both sides of the equation.*	$x + 6 + (-6) = 2 + (-6)$
 *Remove zero pairs from both sides of the mat.*	$x + 0 = 0 + (-4)$
One x-tile is equivalent to 4 red tiles.	$x = -4$

Try This

Use algebra tiles to model and solve each equation.

1. $x + 2 = 5$ **3**
2. $x - 7 = 8$ **15**
3. $x - 5 = 9$ **14**
4. $x + 4 = 7$ **3**

COMMON CORE

1-2 Solving Equations by Adding or Subtracting

CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. … *Also* **CC.9-12.A.REI.3, CC.9-12.A.CED.1***

Objective
Solve one-step equations in one variable by using addition or subtraction.

Vocabulary
equation
solution of an equation

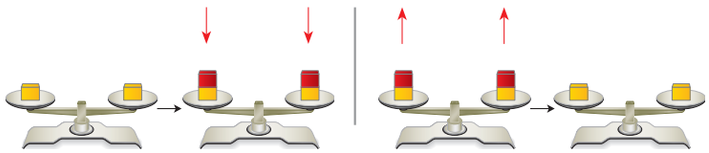
Who uses this?
Athletes can use an equation to estimate their maximum heart rates. (See Example 4.)

An **equation** is a mathematical statement that two expressions are equal. A **solution of an equation** is a value of the variable that makes the equation true.

To find solutions, *isolate the variable*. A variable is isolated when it appears by itself on one side of an equation, and not at all on the other side. Isolate a variable by using inverse operations, which "undo" operations on the variable.

An equation is like a balanced scale. To keep the balance, perform the same operation on both sides.

Inverse Operations
Add *x*. ⟷ Subtract *x*.



EXAMPLE 1 **Solving Equations by Using Addition**

Solve each equation.

A $x - 10 = 4$

$x - 10 = 4$
$+10 \quad +10$
$x = 14$

Since 10 is subtracted from x, add 10 to both sides to undo the subtraction.

Check

$x - 10 = 4$	
$14 - 10$	4
4	4 ✓

To check your solution, substitute 14 for x in the original equation.

B $\frac{2}{5} = m - \frac{1}{5}$

$\frac{2}{5} = m - \frac{1}{5}$
$+\frac{1}{5} \quad +\frac{1}{5}$
$\frac{3}{5} = m$

Since $\frac{1}{5}$ is subtracted from m, add $\frac{1}{5}$ to both sides to undo the subtraction.

Writing Math
Solutions are sometimes written in a *solution set*. For Example 1A, the solution set is {14}. For Example 1B, the solution set is $\left\{\frac{3}{5}\right\}$.

CHECK IT OUT! Solve each equation. Check your answer.

1a. $n - 3.2 = 5.6$ 8.8 **1b.** $-6 = k - 6$ 0 **1c.** $16 = m - 9$ 25

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1-2 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Solve one-step equations in one variable by using addition or subtraction.

Online Edition
Tutorial Videos, Interactivity

Countdown

Power Presentations with PowerPoint®

Warm Up

Simplify.

1. $-\frac{2}{3} + 4\frac{1}{3}$ $3\frac{2}{3}$

2. $-0.51 + (-0.29)$ -0.8

Give the opposite of each number.

3. 8 -8 **4.** $-\frac{2}{3}$ $\frac{2}{3}$

Evaluate each expression for $a = 3$ and $b = -2$.

5. $a + 5$ 8 **6.** $12 - b$ 14

Also available online

Math Humor

Parent: Why do you have that sheet of paper in a bowl of water?

Student: It's my homework. I'm trying to *dissolve* an equation.

COMMON CORE

CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution….

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *Also* **CC.9-12.A.CED.1***

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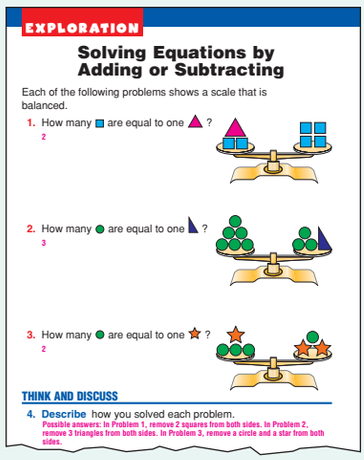
State Resources Online

1 Introduce



Motivate

Place a balance scale with equal weights on each side where all students can see it. Ask them what will happen if you add or subtract an equal amount of weight on both sides. Demonstrate each. Tell the students that equations are like balance scales; both sides must always be kept equal, which means that you must perform the same operation on both sides.

Explorations and answers are provided in your online resources.

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Additional Examples

Example 1

Solve each equation.

A. $y - 8 = 24$ 32

B. $\frac{5}{16} = z - \frac{7}{16}$ $\frac{3}{4}$

Example 2

Solve each equation.

A. $m + 17 = 33$ 16

B. $4.2 = t + 1.8$ 2.4

Example 3

Solve $-\frac{5}{11} + p = -\frac{2}{11}$. $\frac{3}{11}$

Also available online

Questioning Strategies

EXAMPLES 1–2

- What is the goal when solving equations?
- What happens when you add or subtract the same amount on both sides of the equation?
- How do you know when to add and when to subtract?

EXAMPLE 3

- How is this problem like those in **Example 1**? How is it different?

Teaching Tip **Multiple Representations** When discussing the Addition and Subtraction Properties of Equality, give students examples of how subtraction can be written as "adding the opposite." For example, $x - 4 = 6$ is equivalent to $x + (-4) = 6$.

Teaching Tip **Inclusion** Students may have difficulty assigning a variable to represent a quantity in a real-world problem. Reinforce often that a variable represents the unknown. Students can copy a problem and circle the unknown quantity. This is the quantity that should be represented by the variable.

EXAMPLE 2 Solving Equations by Using Subtraction

Solve each equation. Check your answer.

A $x + 7 = 9$

$x + 7 = 9$

$-7 \quad -7$

$x = 2$

Since 7 is added to x, subtract 7 from both sides to undo the addition.

Check

$x + 7 = 9$

$2 + 7 \mid 9$

$9 \mid 9$ ✓

To check your solution, substitute 2 for x in the original equation.

B $0.7 = r + 0.4$

$0.7 = r + 0.4$

$-0.4 \quad -0.4$

$0.3 = r$

Since 0.4 is added to r, subtract 0.4 from both sides to undo the addition.

Check

$0.7 = r + 0.4$

$0.7 \mid 0.3 + 0.4$

$0.7 \mid 0.7$ ✓

To check your solution, substitute 0.3 for r in the original equation.

CHECK IT OUT! **Solve each equation. Check your answer.**

2a. $d + \frac{1}{2} = 1$ $\frac{1}{2}$ **2b.** $-5 = k + 5$ -10 **2c.** $6 + t = 14$ 8

Remember that subtracting is the same as adding the opposite. When solving equations, you will sometimes find it easier to add an opposite to both sides instead of subtracting. For example, this method may be useful when the equation contains negative numbers.

EXAMPLE 3 Solving Equations by Adding the Opposite

Solve $-8 + b = 2$.

$-8 + b = 2$

$+8 \quad +8$

$b = 10$

Since −8 is added to b, add 8 to both sides.

CHECK IT OUT! **Solve each equation. Check your answer.**

3a. $-2.3 + m = 7$ 9.3 **3b.** $-\frac{3}{4} + z = \frac{5}{4}$ 2 **3c.** $-11 + x = 33$ 44

Student to Student — *Zero As a Solution*

Ama Walker
Carson High School

I used to get confused when I got a solution of 0. But my teacher reminded me that 0 is a number just like any other number, so it can be a solution of an equation. Just check your answer and see if it works.

$x + 6 = 6$

$-6 \quad -6$

$x = 0$

Check

$x + 6 = 6$

$0 + 6 \mid 6$

$6 \mid 6$ ✓

RubberBall/Alamy

2 Teach

Guided Instruction

Before solving equations, review how to determine whether a given value is a solution of an equation. Emphasize that when solving equations, you want to find the value of the variable that makes the equation true. So the final answer will be in the form *variable* = *number*. This is why you want to isolate the variable. Discuss how to find a solution by using the inverse operation to "undo" the operation in the equation. Finally, demonstrate how to check the answer by substituting into the original equation.

Reaching All Learners *Through Concrete Manipulatives*

Have one student hold a closed container of pencils in one hand and 5 pencils in the other hand. Have another student hold 12 pencils. Tell the class that both students have the same number of pencils. Ask the class how they can figure out the number of pencils in the container without opening it. Give or take pencils away from each student as suggestions are made. When the class has arrived at an answer, check by counting the number of pencils in the container.

EXAMPLE 4 ***Fitness Application***

A person's maximum heart rate is the highest rate, in beats per minute, that the person's heart should reach. One method to estimate maximum heart rate states that your age added to your maximum heart rate is 220. Using this method, write and solve an equation to find the maximum heart rate of a 15-year-old.

Age	added to	maximum heart rate	is	220.
a	+	r	=	220

$a + r = 220$ *Write an equation to represent the relationship.*

$15 + r = 220$ *Substitute 15 for a. Since 15 is added to r, subtract 15 from both sides to undo the addition.*

$-15 \quad -15$

$r = 205$

The maximum heart rate for a 15-year-old is 205 beats per minute. Since age added to maximum heart rate is 220, the answer should be less than 220. So 205 is a reasonable answer.

CHECK IT OUT!

4. What if...? Use the method above to find a person's age if the person's maximum heart rate is 185 beats per minute.

35 years old

The properties of equality allow you to perform inverse operations, as in the previous examples. These properties say that you can perform the same operation on both sides of an equation.

Know it! Note

Properties of Equality

WORDS	NUMBERS	ALGEBRA
Addition Property of Equality		
You can add the same number to both sides of an equation, and the statement will still be true.	$3 = 3$ $3 + 2 = 3 + 2$ $5 = 5$	$a = b$ $a + c = b + c$
Subtraction Property of Equality		
You can subtract the same number from both sides of an equation, and the statement will still be true.	$7 = 7$ $7 - 5 = 7 - 5$ $2 = 2$	$a = b$ $a - c = b - c$

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Describe how the Addition and Subtraction Properties of Equality are like a balanced scale.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of an equation that can be solved by using the given property, and solve it.

Sam Dudgeon/HMH

COMMON ERROR ALERT

Students sometimes use the operation that appears in an equation rather than using the inverse operation. For example, a student may try to solve $x - 3 = 9$ and get $x = 6$, which is incorrect. Have students first practice naming the operation in the equation and then identifying the inverse operation.

Teaching Tip

Inclusion Continue to use the balance scale analogy throughout the lesson. Draw a simple scale (a line with a triangle under the equal sign) under equations as a reminder to always add or subtract the same amount to both sides to keep the equation in balance.

$$x - 7 = 10$$
$$\triangle$$

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Additional Examples

Example 4

Over 20 years, the population of a town decreased by 275 people to a population of 850. Write and solve an equation to find the original population.

$p - 275 = 850$; 1125

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 4

- What are the variables in this situation?
- What is the unknown variable?
- How can you check that your answer is reasonable?

3 Close

Summarize

Ask students what they would do to each side to solve each of the following equations.

$5 = c - 7$ Add 7.

$-3 + x = 14$ Add 3.

$-14 = h + 6$ Subtract 6.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. Possible answer: If a scale is balanced, then you can add or remove the same weight on both sides without affecting the balance. Similarly, in an equation, the Addition and Subtraction Properties of Equality say that you can add or subtract the same value on both sides without affecting the equality.
2. See Additional Answers.

1-2 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
- **Basic** 21–40, 51–53
- **Average** 21–40, 51, 72, 73
- **Advanced** 21–40, 71–78

If you finished Examples **1–4**
- **Basic** 21–50, 51–66, 64–70
- **Average** 21–57, 59–74
- **Advanced** 21–49, 58–64, 63–78

Homework Quick Check
Quickly check key concepts.
Exercises: 24, 34, 42, 49, 59, 63

Answers

20. Possible answer: the weight of the diamond was reduced by about 50 carats, resulting in a diamond that weighed about 70 carats. So the original weight will be close to 50 + 70 = 120 carats. So 112 carats is a reasonable answer.

49. Possible answer: Luis deposited $500 and now has about $4700. So the original amount will be close to 4700 − 500 = $4200. So $4232 is a reasonable answer.

30. 2.5

35. −3100

39. 0.05

43. 1545

48. −2.8

50. Solution B is incorrect. The Addition Property of Equality was used and $x + 25$ is what truly appears on the left side of the equation.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 20, 49, 58–60, 64–65, 70

Construct viable arguments and critique the reasoning of others. Exercises 50, 66–67

Model with mathematics. Exercises 61–63

Learn It Online
State Resources Online

GUIDED PRACTICE

1. **Vocabulary** Will the *solution of an equation* such as $x - 3 = 9$ be a variable or a number? Explain. Possible answer: The solution of an equation is a number. It is a value for the variable that works in the equation.

Solve each equation. Check your answer.

SEE EXAMPLE 1

2. $s - 5 = 3$ 8
3. $17 = w - 4$ 21
4. $k - 8 = -7$ 1
5. $x - 3.9 = 12.4$ 16.3
6. $8.4 = y - 4.6$ 13
7. $\frac{3}{8} = t - \frac{1}{8}$ $\frac{1}{2}$

SEE EXAMPLE 2

8. $t + 5 = -25$ −30
9. $9 = s + 9$ 0
10. $42 = m + 36$ 6
11. $2.8 = z + 0.5$ 2.3
12. $b + \frac{2}{3} = 2$ $\frac{4}{3}$
13. $n + 1.8 = 3$ 1.2

SEE EXAMPLE 3

14. $-10 + d = 7$ 17
15. $20 = -12 + v$ 32
16. $-46 + q = 5$ 51
17. $2.8 = -0.9 + y$ 3.7
18. $-\frac{2}{3} + c = \frac{2}{3}$ $\frac{4}{3}$
19. $-\frac{5}{6} + p = 2$ $\frac{17}{6}$

SEE EXAMPLE 4

20. **Geology** In 1673, the Hope diamond was reduced from its original weight by about 45 carats, resulting in a diamond weighing about 67 carats. Write and solve an equation to find how many carats the original diamond weighed. Show that your answer is reasonable. $w - 45 = 67$; 112 carats

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
21–30	1
31–40	2
41–48	3
49	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Solve each equation. Check your answer.

21. $1 = k - 8$ 9
22. $u - 15 = -8$ 7
23. $x - 7 = 10$ 17
24. $-9 = p - 2$ −7
25. $\frac{3}{7} = p - \frac{1}{7}$ $\frac{4}{7}$
26. $q - 0.5 = 1.5$ 2
27. $6 = t - 4.5$ 10.5
28. $4\frac{2}{3} = r - \frac{1}{3}$ 5
29. $6 = x - 3$ 9
30. $1.75 = k - 0.75$
31. $19 + a = 19$ 0
32. $4 = 3.1 + y$ 0.9
33. $m + 20 = 3$ −17
34. $-12 = c + 3$ −15
35. $v + 2300 = -800$
36. $b + 42 = 300$ 258
37. $3.5 = n + 4$ −0.5
38. $b + \frac{1}{2} = \frac{1}{2}$ 0
39. $x + 5.34 = 5.39$
40. $2 = d + \frac{1}{4}$ $\frac{7}{4}$
41. $-12 + f = 3$ 15
42. $-9 = -4 + g$ −5
43. $-1200 + j = 345$
44. $90 = -22 + a$ 112
45. $26 = -4 + y$ 30
46. $1\frac{3}{4} = -\frac{1}{4} + w$ 2
47. $-\frac{1}{6} + h = \frac{1}{6}$ $\frac{1}{3}$
48. $-5.2 + a = -8$

49. **Finance** Luis deposited $500 into his bank account. He now has $4732. Write and solve an equation to find how much was in his account before the deposit. Show that your answer is reasonable. $a + 500 = 4732$; $4232

50. ERROR ANALYSIS Below are two possible solutions to $x + 12.5 = 21.6$. Which is incorrect? Explain the error.

A

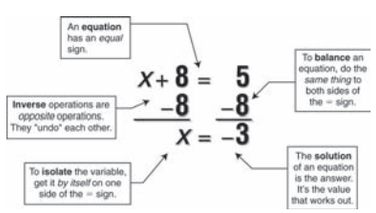
$x + 12.5 = 21.6$
$-12.5 \quad -12.5$
$x = 9.1$

B

$x + 12.5 = 21.6$
$+12.5 \quad +12.5$
$x = 34.1$

READING STRATEGIES

To solve equations, you must know many mathematical words and phrases. Look at the diagram below to help you better understand this vocabulary.



Answer each of the following.

1. What is the **inverse** operation of subtraction? addition
2. If you add 5 to the right side of an equation, how do you keep the equation **balanced**? Add 5 to the left side.
3. How do you **isolate** the variable in the equation $p - 4 = 12$? Add 4 to both sides.

Solve each equation.

4. $m - 9 = 4$ $m = 13$
5. $13 = g + 8$ $g = 5$
6. $k + 5.8 = 2.8$ $k = -3$
7. $-3 = t + 12$ $t = -15$
8. $-30 = y - 32$ $y = 2$
9. $-2 + h = 9$ $h = 11$

RETEACH

Use counters to model solving equations.

Solve $x + 2 = 5$.

Using counters	Using numbers
$x +$ ○○ = ○○○○○	$x + 2 = 5$
$x +$ ▨▨ = ○○○○○	$x + 2 = 5$ $-2 \quad -2$
$x +$ 0 = ○○○	$x + 0 = 3$
$x =$ ○○○	$x = 3$
Check: ○○ + ○○○ = ○○○○○	Check: $x + 2 = 5$ $3 + 2 \stackrel{?}{=} 5$ $5 \stackrel{?}{=} 5$ ✓

Solve the following by drawing counters. Check your answers.

1. $x + 1 = 5$ $x +$ ▨ = ○○○○ 4
2. $7 = x + 2$ ○○○○○○○ = $x +$ ▨▨ 5

Solve each equation. Check your answers.

3. $x + 4 = 12$ 8
4. $21 = x + 2$ 19
5. $x + 3 = 8$ 5

Write an equation to represent each relationship. Then solve the equation.

51. Ten less than a number is equal to 12. $x - 10 = 12$; $x = 22$

52. A number decreased by 13 is equal to 7. $x - 13 = 7$; $x = 20$

53. Eight more than a number is 16. $x + 8 = 16$; $x = 8$

54. A number minus 3 is –8. $x - 3 = -8$; $x = -5$

55. The sum of 5 and a number is 6. $5 + x = 6$; $x = 1$

56. Two less than a number is –5. $x - 2 = -5$; $x = -3$

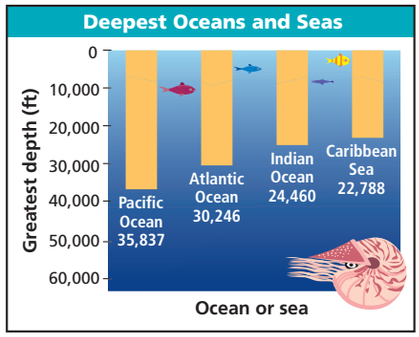
57. The difference of a number and 4 is 9. $x - 4 = 9$; $x = 13$

LINK

Geology

The ocean depths are home to many odd-looking creatures. The anglerfish pictured above, known as the common black devil, may appear menacing but reaches a maximum length of only about 5 inches.

58. Geology The sum of the Atlantic Ocean's average depth (in feet) and its greatest depth is 43,126. Use the information in the graph to write and solve an equation to find the average depth of the Atlantic Ocean. Show that your answer is reasonable.

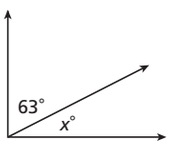


59. School Helene's marching band needs money to travel to a competition. Band members have raised $560. They need to raise a total of $1680. Write and solve an equation to find how much more they need. Show that your answer is reasonable. $m + 560 = 1680$; $1120

60. Economics When you receive a loan to make a purchase, you often must make a down payment in cash. The amount of the loan is the purchase cost minus the down payment. Riva made a down payment of $1500 on a used car. She received a loan of $2600. Write and solve an equation to find the cost of the car. Show that your answer is reasonable. $c - 1500 = 2600$; $4100

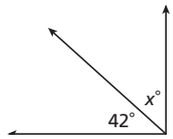
Geometry **The angles in each pair are complementary. Write and solve an equation to find each value of *x*. (*Hint:* The measures of complementary angles add to 90°.)**

61.



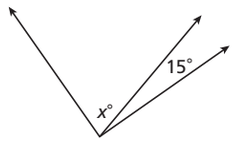
$63 + x = 90$; $x = 27$

62.



$42 + x = 90$; $x = 48$

63.



$x + 15 = 90$; $x = 75$

MULTI-STEP TEST PREP

64. *Rates* are often used to describe how quickly something is moving or changing.

a. A wildfire spreads at a rate of 1000 acres per day. How many acres will the fire cover in 2 days? Show that your answer is reasonable. 2000 acres

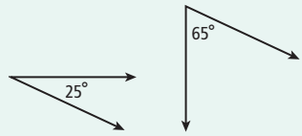
b. How many acres will the fire cover in 5 days? Explain how you found your answer. 5000 acres; multiply 5 days by 1000 acres per day.

c. Another wildfire spread for 7 days and covered a total of 780 square miles. How can you estimate the number of square miles the fire covered per day? Divide 780 by 7.

(t) Dante Fenolio / Photo Researchers, Inc.; (b) ©Index Stock/Alamy

Teaching Tip

Geometry In **Exercises 61–63,** students are asked to write equations for complementary angles. Two angles are complementary if their sum is 90°. Remind students that complementary angles need not be adjacent. The angles below are complementary.



MULTI-STEP TEST PREP

Exercise 64 involves using rates to predict how quickly a fire will spread and how much land it will cover. This exercise prepares students for the Multi-Step Test Prep.

Answers

58. $a + 30{,}246 = 43{,}126$; 12,880; Possible answer: the greatest depth is about 30,000 feet. The sum of this number and the average is about 43,000 feet. So the average will be close to 43,000 – 30,000 = 13,000. So 12,880 ft is a reasonable answer.

59. Possible answer: The band needs about $1700 and they have raised about $600. So the amount they need will be close to 1700 – 600 = 1100. So $1120 is a reasonable answer.

60. Possible answer: the total cost should be more than the down payment and more than the loan amount, so $4100 is a reasonable answer.

64a. For explanation, see Additional Answers.

PRACTICE A

PRACTICE C

PRACTICE B

Solve each equation. Check your answers.

1. $g - 7 = 15$ — $g = 22$
2. $t + 4 = 6$ — $t = 2$
3. $13 = m - 7$ — $m = 20$
4. $x + 3.4 = 9.1$ — $x = 5.7$
5. $n - \frac{3}{8} = \frac{1}{8}$ — $n = \frac{1}{2}$
6. $p - \frac{1}{3} = \frac{2}{3}$ — $p = 1$
7. $-6 + k = 32$ — $k = 38$
8. $7 = w + 9.3$ — $w = -2.3$
9. $8 = r + 12$ — $r = -4$
10. $y - 57 = -40$ — $y = 17$
11. $-5.1 + b = -7.1$ — $b = -2$
12. $a + 15 = 15$ — $a = 0$

13. Marietta was given a raise of $0.75 an hour, which brought her hourly wage to $12.25. Write and solve an equation to determine Marietta's hourly wage before her raise. Show that your answer is reasonable.
$x + 0.75 = 12.25$; $11.50; Marietta received a raise, so her previous wage should be less than $12.25.

14. Brad grew $4\frac{1}{4}$ inches this year and is now $56\frac{7}{8}$ inches tall. Write and solve an equation to find Brad's height at the start of the year. Show that your answer is reasonable.
$x + 4\frac{1}{4} = 56\frac{7}{8}$; $52\frac{5}{8}$ in.; Brad's height increased, so his previous height should be less than $56\frac{7}{8}$ inches.

15. Heather finished a race in 58.4 seconds, which was 2.6 seconds less than her practice time. Write and solve an equation to find Heather's practice time. Show that your answer is reasonable.
$x - 2.6 = 58.4$; 61 s; Heather's race time was less than her practice time, so her practice time should be greater than 58.4 s.

16. The radius of Earth is 6378.1 km, which is 2981.1 km longer than the radius of Mars. Write and solve an equation to determine the radius of Mars. Show that your answer is reasonable.
$x + 2981.1 = 6378.1$; 3397 km; Earth is larger than Mars, so the radius of Mars should be less than 6378.1 km.

PROBLEM SOLVING

Write the correct answer.

1. Michelle withdrew $120 from her bank account. She now has $3345 in her account. Write and solve an equation to find how much money *m* was in her account before she made the withdrawal.
$m - 120 = 3345$; $3465

2. Max lost 23 pounds while on a diet. He now weighs 184 pounds. Write and solve an equation to find his initial weight *w*.
$w - 23 = 184$; 207 lbs

3. Earth takes 365 days to orbit the Sun. Mars takes 687 days. Write and solve an equation to find how many more days *d* Mars takes than Earth to orbit the Sun.
$365 + d = 687$; 322 days

4. In 1990, 53.4% of commuters took public transportation in New York City, which was 19.9% greater than the percentage in San Francisco. Write and solve an equation to find what percentage of commuters *p* took public transportation in San Francisco.
$p + 19.9 = 53.4$; 33.5%

Use the circle graph below to answer questions 5–7. Select the best answer. The circle graph shows the colors for SUVs as percents of the total number of SUVs manufactured in 2000 in North America.

5. The percent of silver SUVs increased by 7.9% between 1998 and 2000. If *x*% of SUVs were silver in 1998, which equation represents this relationship?
A $x + 7.9 = 14.1$ C $7.9x = 14.1$
B $x - 7.9 = 14.1$ D $7.9 - x = 14.1$

6. Solve the equation from problem 5. What is the value of *x*?
F 1.8 H 7.1
G 6.2 J 22

7. The sum of the percents of dark red SUVs and white SUVs was 26.3%. What was the percent of dark red SUVs?
A 2.3% C 12.2%
B 3.2% D 18%

Percent of SUVs by Color

CHALLENGE

The function of an odometer is to measure the distance that a vehicle has traveled, either in miles or in kilometers. When you use an odometer for this purpose, you can answer the question, "How far have I traveled?" The reasoning that you used to answer the question can be applied to stating and solving new related problems.

Answer the following questions.

1. You are on a trip traveling from town A to town B. When you start the trip, your odometer reading is 37,538 miles. When you get to town B, your odometer reading is 37,781.
a. Write an equation relating your initial reading, distance traveled, and final reading. Let *x* represent the distance traveled. $37{,}538 + x = 37{,}781$
b. Find the distance traveled from town A to town B. 243 miles

2. Let *s* represent the initial reading, *d* represent the distance traveled, and *e* represent the final reading. Write an addition equation relating *s*, *d*, and *e*.
$s + d = e$

3. Using your equation from Exercise 2, solve for the specified variable.
a. $s = e - d$ b. $d = e - s$ c. $e = s + d$

4. Use one of the equations that you wrote in Exercise 3 and the given values to find the value of the third variable.
a. $s = 27{,}281$ and $e = 28{,}978$ — $d = 1697$
b. $e = 17{,}349$ and $d = 197$ — $s = 17{,}152$
c. $s = 62{,}979$ and $d = 798$ — $e = 63{,}777$
d. $s = 69{,}876$ and $e = 70{,}987$ — $d = 1111$

The water level in a storage tank is currently $10\frac{1}{2}$ feet.

5. Water is drained from the tank, and the level drops $2\frac{5}{6}$ feet. Find the new water level.
$7\frac{2}{3}$ feet

6. Let *o* represent the original water level, *d* represent the change in water level, and *f* represent the final water level. Write three equations relating *o*, *d*, and *f*.
$f = o + d$, $o = f - d$, and $d = f - o$

7. The water level rises $1\frac{3}{4}$ feet from $7\frac{2}{3}$ feet. Choose an equation from Exercise 6 and find the new water level.
$9\frac{5}{12}$ feet

TEST PREP DOCTOR In **Exercise 68,** 32 is being subtracted from x, so students should look for a situation that shows a decrease of 32. Choice **A** has a withdrawal of 32.

If a student selects **H** in **Exercise 69,** he or she may have subtracted 18 from both sides of the equation.

65. Statistics The range of a set of scores is 28, and the lowest score is 47. Write and solve an equation to find the highest score. (*Hint:* In a data set, the range is the difference between the highest and the lowest values.) Show that your answer is reasonable. $h - 47 = 28$; 75

66. Write About It Describe a real-world situation that can be modeled by $x + 5 = 25$. Tell what the variable represents in your situation. Then solve the equation and tell what the solution means in the context of your problem.

67. Critical Thinking Without solving, tell whether the solution of $-3 + z = 10$ will be greater than 10 or less than 10. Explain. Possible answer: greater than 10 because you will add a positive number to both sides

TEST PREP

68. Which situation is best represented by $x - 32 = 8$?

(A) Logan withdrew \$32 from her bank account. After her withdrawal, her balance was \$8. How much was originally in her account?

(B) Daniel has 32 baseball cards. Joseph has 8 fewer baseball cards than Daniel. How many baseball cards does Joseph have?

(C) Room A contains 32 desks. Room B has 8 fewer desks. How many desks are in Room B?

(D) Janelle bought a bag of 32 craft sticks for a project. She used 8 craft sticks. How many craft sticks does she have left?

69. For which equation is $a = 8$ a solution?

(F) $15 - a = 10$ (G) $10 + a = 23$ (H) $a - 18 = 26$ (J) $a + 8 = 16$

70a. $g - 18 = 22$
b. \$40

70. Short Response Julianna used a gift card to pay for an \$18 haircut. The remaining balance on the card was \$22.

a. Write an equation that can be used to determine the original value of the card.

b. Solve your equation to find the original value of the card.

CHALLENGE AND EXTEND

Solve each equation. Check your answer.

$-\frac{12}{5}$ **71.** $3\frac{1}{5} + b = \frac{4}{5}$ **72.** $x - \frac{7}{4} = \frac{2}{3}$ $\frac{29}{12}$ **73.** $x + \frac{7}{4} = \frac{2}{3}$ $-\frac{13}{12}$ **74.** $x - \frac{4}{9} = \frac{4}{9}$ $\frac{8}{9}$

75. If $p - 4 = 2$, find the value of $5p - 20$. 10

76. If $t + 6 = 21$, find the value of $-2t$. −30

77. If $x + 3 = 15$, find the value of $18 + 6x$. 90

78. If $2 + n = -11$, find the value of $6n$. −78

Journal

Have students explain the Addition and Subtraction Properties of Equality, and how they are used to solve equations.

ALTERNATIVE ASSESSMENT

Have students write and solve four equations: two that can be solved using the Addition Property of Equality, and two that can be solved using the Subtraction Property of Equality.

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1-2 Lesson Quiz

Solve each equation.

1. $r - 4 = -8$ −4

2. $\frac{5}{12} = s - \frac{11}{12}$ $\frac{16}{12}$, or $\frac{4}{3}$

3. $m + 13 = 58$ 45

4. $0.75 = n + 0.6$ 0.15

5. $-5 + c = 22$ 27

6. This year a high school had 578 sophomores enrolled. This is 89 less than the number enrolled last year. Write and solve an equation to find the number of sophomores enrolled last year.
$s - 89 = 578$; $s = 667$

Also available online

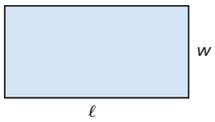
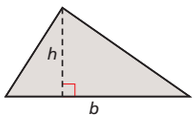
Answers

65. Possible answer: the range is about 30 and the lowest score is about 50. So the highest score will be close to 30 + 50 = 80. So 75 is a reasonable answer.

66. Possible answer: If you add 5 yr to Sue's age, you get her cousin's age. Her cousin is 25. How old is Sue? x represents Sue's age. $x = 20$; this means Sue is 20 years old.

Area of Composite Figures

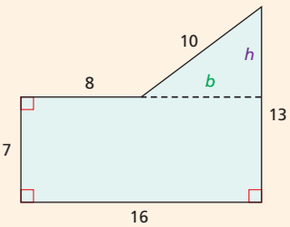
Review the area formulas for squares, rectangles, and triangles in the table below.

Squares	Rectangles	Triangles
		
$A = s^2$	$A = \ell w$	$A = \frac{1}{2}bh$

A *composite figure* is a figure that is composed of basic shapes. You can divide composite figures into combinations of squares, rectangles, and triangles to find their areas.

Example

Find the area of the figure shown.



Divide the figure into a rectangle and a right triangle. Notice that you do not know the base or the height of the triangle. Use b and h to represent these lengths.

The bottom of the rectangle is 16 units long; the top of the rectangle is 8 units long plus the base of the triangle. Use this information to write and solve an equation.

$$\begin{array}{rcl} b + 8 & = & 16 \\ \underline{-\,8} & & \underline{-\,8} \\ b & = & 8 \end{array}$$

The right side of the figure is 13 units long: 7 units from the rectangle plus the height of the triangle. Use this information to write and solve an equation.

$$\begin{array}{rcl} h + 7 & = & 13 \\ \underline{-\,7} & & \underline{-\,7} \\ h & = & 6 \end{array}$$

The area of the figure is the sum of the areas of the rectangle and the triangle.

Area of rectangle ↓ + Area of triangle ↓

$$A = \ell w + \frac{1}{2}bh$$

$$A = 16(7) + \frac{1}{2}(8)(6)$$

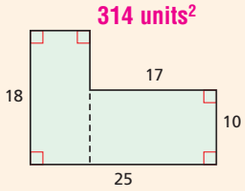
$$A = 112 + 24$$

$$A = 136 \text{ square units}$$

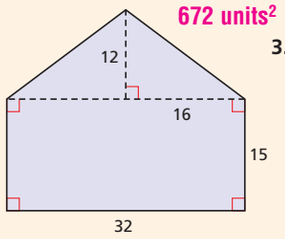
Try This

Find the area of each composite figure.

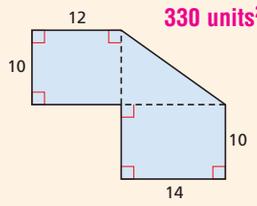
1. **314 units²**



2. **672 units²**



3. **330 units²**



Organizer

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Apply algebra skills to finding areas of geometric figures.

Online Edition

Teach

Remember

Students review and apply area formulas for geometric figures.

Visual Have students decide how to divide the composite figure and then draw and label each part separately.

Close

Assess

Have students divide the figure in **Exercise 1** differently than the first time they found the area. Have them show algebraically that the area is still the same.

Learn It Online
State Resources Online

1-3 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Solve one-step equations in one variable by using multiplication or division.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Evaluate each expression.

1. $(-7)(2.8)$ -19.6
2. $0.96 \div 6$ 0.16
3. $(-9)(-9)$ 81
4. $\left(\frac{5}{6}\right)\left(\frac{6}{5}\right)$ 1
5. $\left(\frac{8}{15}\right)\left(\frac{5}{4}\right)$ $\frac{2}{3}$
6. $\left(\frac{3}{4}\right)(2.4)$ 1.8

Also available online

Math Humor

Teacher: Why don't you have your homework today?

Student: I divided by zero and the paper vanished into thin air!

COMMON CORE

CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution….

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *Also* **CC.9-12.A.CED.1***

Learn It Online
State Resources Online

COMMON CORE

1-3 Solving Equations by Multiplying or Dividing

CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. … *Also* **CC.9-12.A.REI.3, CC.9-12.A.CED.1***

Objective
Solve one-step equations in one variable by using multiplication or division.

Who uses this?
Pilots can make quick calculations by solving one-step equations. (See Example 4.)

Solving an equation that contains multiplication or division is similar to solving an equation that contains addition or subtraction. Use inverse operations to undo the operations on the variable.

Remember that an equation is like a balanced scale. To keep the balance, whatever you do on one side of the equation, you must also do on the other side.

Inverse Operations
Multiply by x. ↔ Divide by x.

EXAMPLE 1 Solving Equations by Using Multiplication

Solve each equation. Check your answer.

A $-4 = \frac{k}{-5}$

$(-5)(-4) = (-5)\left(\frac{k}{-5}\right)$ *Since k is divided by −5, multiply both sides by −5 to undo the division.*

$20 = k$

Check $-4 = \frac{k}{-5}$ *To check your solution, substitute 20 for k in the original equation.*

-4	$\frac{20}{-5}$
-4	-4 ✓

B $\frac{m}{3} = 1.5$

$(3)\left(\frac{m}{3}\right) = (3)(1.5)$ *Since m is divided by 3, multiply both sides by 3 to undo the division.*

$m = 4.5$

Check $\frac{m}{3} = 1.5$ *To check your solution, substitute 1.5 for m in the original equation.*

$\frac{4.5}{3}$	1.5
1.5	1.5 ✓

CHECK IT OUT!

Solve each equation. Check your answer.

1a. $\frac{p}{5} = 10$ **50** **1b.** $-13 = \frac{y}{3}$ **−39** **1c.** $\frac{c}{8} = 7$ **56**

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1 Introduce

EXPLORATION

Solving Equations by Multiplying or Dividing

Complete the tables by evaluating the expressions for the given values of x.

1.

x	4x
1	4
4	16
7	28
9	36

2.

x	$\frac{x}{3}$
3	1
12	4
18	6
30	10

Complete the tables by finding the values of x.

3.

x	4x
2	8
5	20
6	24
12	48

4.

x	$\frac{x}{3}$
6	2
9	3
15	5
45	15

THINK AND DISCUSS

5. **Explain** how you evaluated the expressions in Problems 1 and 2. In Problem 1, multiply the value of x by 4. In Problem 2, divide the value of x by 3.
6. **Explain** how you found the values of x in Problems 3 and 4. In Problem 3, divide the values in the right column by 4. In Problem 4, multiply the values in the right column by 3.

Motivate

Ask students what operation is being performed on each variable, and by what number.

$5x = 10$ multiplication by 5

$12 = \frac{p}{6}$ division by 6

Tell students that in this lesson, they will "undo" these operations to solve the equations, just as they had to "undo" addition and subtraction in the previous lesson.

Explorations and answers are provided in your online resources.

EXAMPLE 2 Solving Equations by Using Division

Solve each equation. Check your answers.

A $7x = 56$

$\frac{7x}{7} = \frac{56}{7}$ — *Since x is multiplied by 7, divide both sides by 7 to undo the multiplication.*

$x = 8$

Check

$7x = 56$	
$7(8)$	56
56	56 ✓

To check your solution, substitute 8 for x in the original equation.

B $13 = -2w$

$\frac{13}{-2} = \frac{-2w}{-2}$ — *Since w is multiplied by −2, divide both sides by −2 to undo the multiplication.*

$-6.5 = w$

Check

$13 = -2w$	
13	$-2(-6.5)$
13	13 ✓

To check your solution, substitute −6.5 for w in the original equation.

CHECK IT OUT! Solve each equation. Check your answer.

2a. $16 = 4c$ 4 **2b.** $0.5y = -10$ −20 **2c.** $15k = 75$ 5

Remember that dividing is the same as multiplying by the reciprocal. When solving equations, you will sometimes find it easier to multiply by a reciprocal instead of dividing. This is often true when an equation contains fractions.

EXAMPLE 3 Solving Equations That Contain Fractions

Solve each equation.

A $\frac{5}{9}v = 35$

$\left(\frac{9}{5}\right)\frac{5}{9}v = \left(\frac{9}{5}\right)35$ — *The reciprocal of $\frac{5}{9}$ is $\frac{9}{5}$. Since v is multiplied by $\frac{5}{9}$, multiply both sides by $\frac{9}{5}$.*

$v = 63$

B $\frac{5}{2} = \frac{4y}{3}$

$\frac{5}{2} = \frac{4y}{3}$

$\frac{5}{2} = \frac{4}{3}y$ — *$\frac{4y}{3}$ is the same as $\frac{4}{3}y$.*

$\left(\frac{3}{4}\right)\frac{5}{2} = \left(\frac{3}{4}\right)\frac{4}{3}y$ — *The reciprocal of $\frac{4}{3}$ is $\frac{3}{4}$. Since y is multiplied by $\frac{4}{3}$, multiply both sides by $\frac{3}{4}$.*

$\frac{15}{8} = y$

CHECK IT OUT! Solve each equation. Check your answer.

3a. $-\frac{1}{4} = \frac{1}{5}b$ $-\frac{5}{4}$ **3b.** $\frac{4j}{6} = \frac{2}{3}$ 1 **3c.** $\frac{1}{6}w = 102$ 612

COMMON ERROR ALERT

When solving problems using division, some students may automatically divide the larger number by the smaller number every time. For example, a student may try to solve $6x = 3$ by dividing both sides by 3. Remind students that they must isolate the variable, and that the variable is being multiplied by 6, not by 3.

Power Presentations with PowerPoint®

Additional Examples

Example 1

Solve each equation.

A. $-8 = \frac{j}{3}$ −24

B. $\frac{n}{6} = 2.8$ 16.8

Example 2

Solve each equation.

A. $9y = 108$ 12

B. $-4.8 = -6v$ 0.8

Example 3

Solve each equation.

A. $\frac{5}{6}w = -20$ −24

B. $\frac{3}{16} = \frac{1}{8}z$ $\frac{3}{2}$, or $1\frac{1}{2}$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- What operation is used to isolate the variable? Why?

EXAMPLE 2

- What operation is used to isolate the variable? Why?
- How do you know which number to divide both sides by?

EXAMPLE 3

- What is the product of any number and its reciprocal? How does that help you solve equations?

2 Teach

Guided Instruction

Review multiplication and division of signed numbers, as well as how to find the reciprocal of a number. Show how to solve equations by using multiplication and division to undo the operation shown. Remind students that the coefficient of a single variable is 1, so the coefficient of $-x$ is -1.

Teaching Tip **Reading Math** Remind students that $-x = 3$ can be read as "the opposite of x equals 3."

ENGLISH LANGUAGE LEARNERS

Reaching All Learners

Through Visual Cues

Have students create a chart they can use as a reference on class work or homework.

When Solving Equations	
I see...	**I use...**
Addition	Subtraction
Subtraction	Addition
Multiplication	Division
Division	Multiplication

Power Presentations with PowerPoint®

Additional Examples

Example 4

Ciro puts $\frac{1}{4}$ of the money he earns from mowing lawns into a college education fund. This year Ciro added $285 to his college education fund. Write and solve an equation to find how much money Ciro earned mowing lawns this year. $\frac{1}{4}m = 285$; $1140

Also available online

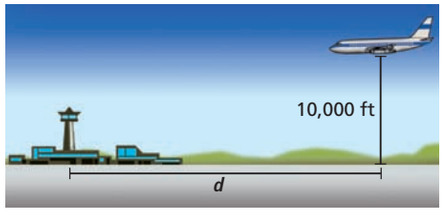
Questioning Strategies

EXAMPLE 4

- What are the variables in the situation?
- What operation is needed to solve the equation?

Inclusion The expression $7x$ means 7 of whatever quantity x represents. To help students understand this, give a real-world example. If 7 CDs cost $56, but you only want to buy 1 CD, how would you find the cost?

EXAMPLE 4 ***Aviation Application***



The distance in miles from the airport that a plane should begin descending, divided by 3, equals the plane's height above the ground in thousands of feet. If a plane is 10,000 feet above the ground, write and solve an equation to find the distance at which the pilot should begin descending.

Caution!

The equation uses the plane's height above the ground in *thousands* of feet. So substitute 10 for h, not 10,000.

Distance | divided by 3 | equals | height in thousands of feet.

$\frac{d}{3} = h$ — *Write an equation to represent the relationship.*

$\frac{d}{3} = 10$ — *Substitute 10 for h. Since d is divided by 3, multiply both sides by 3 to undo the division.*

$(3)\frac{d}{3} = (3)10$

$d = 30$

The pilot should begin descending 30 miles from the airport.

CHECK IT OUT! **4. What if...?** A plane began descending 45 miles from the airport. Use the equation above to find how high the plane was flying when the descent began. **15,000 ft**

You have now used four properties of equality to solve equations. These properties are summarized in the box below.

Know it! Note

Properties of Equality

WORDS	NUMBERS	ALGEBRA
Addition Property of Equality		
You can add the same number to both sides of an equation, and the statement will still be true.	$3 = 3$ $3 + 2 = 3 + 2$ $5 = 5$	$a = b$ $a + c = b + c$
Subtraction Property of Equality		
You can subtract the same number from both sides of an equation, and the statement will still be true.	$7 = 7$ $7 - 5 = 7 - 5$ $2 = 2$	$a = b$ $a - c = b - c$
Multiplication Property of Equality		
You can multiply both sides of an equation by the same number, and the statement will still be true.	$6 = 6$ $6(3) = 6(3)$ $18 = 18$	$a = b$ $ac = bc$
Division Property of Equality		
You can divide both sides of an equation by the same nonzero number, and the statement will still be true.	$8 = 8$ $\frac{8}{4} = \frac{8}{4}$ $2 = 2$	$a = b$ $(c \neq 0)$ $\frac{a}{c} = \frac{b}{c}$

3 Close

Summarize

Ask students to explain which property of equality should be used to solve each equation.

$-4y = 20$ — Div. Prop. of Eq.

$-7 + f = -\frac{1}{2}$ — Add. Prop. of Eq.

$15 = p + 4$ — Subt. Prop. of Eq.

$\frac{x}{33} = -9$ — Mult. Prop. of Eq.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

THINK AND DISCUSS

MATHEMATICAL PRACTICES

1. Tell how the Multiplication and Division Properties of Equality are similar to the Addition and Subtraction Properties of Equality.

Know it! Note

2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of an equation that can be solved by using the given property, and solve it.

1-3 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

Solve each equation. Check your answer.

SEE EXAMPLE 1

1. $\frac{k}{4} = 8$ **32**
2. $\frac{z}{3} = -9$ **−27**
3. $-2 = \frac{w}{-7}$ **14**
4. $6 = \frac{t}{-5}$ **−30**
5. $\frac{g}{1.9} = 10$ **19**
6. $2.4 = \frac{b}{5}$ **12**

SEE EXAMPLE 2

7. $4x = 28$ **7**
8. $-64 = 8c$ **−8**
9. $-9j = -45$ **5**
10. $84 = -12a$ **−7**
11. $4m = 10$ **2.5**
12. $2.8 = -2h$ **−1.4**

SEE EXAMPLE 3

13. $\frac{1}{2}d = 7$ **14**
14. $15 = \frac{5}{6}f$ **18**
15. $\frac{2}{3}s = -6$ **−9**
16. $9 = -\frac{3}{8}r$ **−24**
17. $\frac{1}{10} = \frac{4}{5}y$ $\frac{1}{8}$
18. $\frac{1}{4}v = -\frac{3}{4}$ **−3**

SEE EXAMPLE 4

19. **Recreation** The Baseball Birthday Batter Package at a minor league ballpark costs $192. The package includes tickets, drinks, and cake for a group of 16 children. Write and solve an equation to find the cost per child. **16*c* = 192; $12**
20. **Nutrition** An orange contains about 80 milligrams of vitamin C, which is 10 times as much as an apple contains. Write and solve an equation to find the amount of vitamin C in an apple. **80 = 10*a*; 8 mg**

PRACTICE AND PROBLEM SOLVING

Solve each equation. Check your answer.

21. $\frac{x}{2} = 12$ **24**
22. $-40 = \frac{b}{5}$ **−200**
23. $-\frac{j}{6} = 6$ **−36**
24. $-\frac{n}{3} = -4$ **12**
25. $-\frac{q}{5} = 30$ **−150**
26. $1.6 = \frac{d}{3}$ **4.8**
27. $\frac{v}{10} = 5.5$ **55**
28. $\frac{h}{8.1} = -4$ **−32.4**
29. $5t = -15$ **−3**
30. $49 = 7c$ **7**
31. $-12 = -12u$ **1**
32. $-7m = 63$ **−9**
33. $-52 = -4c$ **13**
34. $11 = -2z$ **−5.5**
35. $5f = 1.5$ **0.3**
36. $-8.4 = -4n$ **2.1**

Answers to *Think and Discuss*

1. Possible answer: All 4 properties tell you that if you perform the same operation on both sides of an equation, the equality will still hold.
2. See Additional Answers.

1-3 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**

Basic	21–36, 52–55
Average	21–36, 52, 54, 61–64
Advanced	22–36 even, 52–56, 61–64, 88–91

If you finished Examples **1–4**

Basic	21–57, 59–75, 77–81
Average	21–47, 48–54 even, 56–76, 77–88
Advanced	22–42, 48–54 even, 58–94 even

Homework Quick Check
Quickly check key concepts.
Exercises: 22, 32, 38, 46, 50, 74

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 19–20, 45–46, 56–60, 65, 74–75

Construct viable arguments and critique the reasoning of others. Exercises 47, 76

Look for and express regularity in repeated reasoning. Exercise 94

Learn It Online
State Resources Online

Teaching Tip **Earth Science Link** In **Exercise 46,** a relationship is given between people's weight on Earth and their weight on the Moon. Students may also be interested to know that people's weight on Pluto is about $\frac{7}{100}$ their weight on Earth, and their weight on Jupiter is about twice their weight on Earth.

Teaching Tip **Geometry** In **Exercises 48–51,** students must write an equation for the perimeter of a square. All four sides of a square have equal length, so the formula can be written in the form $4s = P$.

Answers

46. Possible answer: the person's weight on the moon is about 15 lb. Estimate that the answer will be close to $15 \cdot 6 = 90$. So 96 lb is reasonable.

59. Possible answer: Dion's calls cost \$0.05/min. If he talked for 300 min, the cost would be $300(0.05) = \$15$. This is close to the cost given in the problem, \$13.80. So 276 min is reasonable.

60. Possible answer: the amount of caffeine in the soft drink should be about $\frac{200}{5} = 40$ mg. So 37 mg is reasonable.

Independent Practice

For Exercises	See Example
21–28	1
29–36	2
37–44	3
45	4

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

46. $\frac{1}{6}w = 16$; 96 lb

Solve each equation. Check your answer.

37. $\frac{5}{2}k = 5$ **2**
38. $-9 = \frac{3}{4}d$ **−12**
39. $-\frac{5}{8}b = 10$ **−16**
40. $-\frac{4}{5}g = -12$ **15**
41. $\frac{4}{7}t = -2$ **−3.5**
42. $-\frac{4}{5}p = \frac{2}{3}$ **$-\frac{5}{6}$**
43. $\frac{2}{3} = -\frac{1}{3}q$ **−2**
44. $-\frac{5}{8} = -\frac{3}{4}a$ **$\frac{5}{6}$**

45. **Finance** After taxes, Alexandra's take-home pay is $\frac{7}{10}$ of her salary before taxes. Write and solve an equation to find Alexandra's salary before taxes for the pay period that resulted in \$392 of take-home pay. **$\frac{7}{10}s = 392$; \$560**

46. **Earth Science** Your weight on the Moon is about $\frac{1}{6}$ of your weight on Earth. Write and solve an equation to show how much a person weighs on Earth if he weighs 16 pounds on the Moon. How could you check that your answer is reasonable?

47. **ERROR ANALYSIS** For the equation $\frac{x}{3} = 15$, a student found the value of x to be 5. Explain the error. What is the correct answer? **The student divided both sides by 3 instead of multiplying both sides by 3; 45.**

Geometry **The perimeter of a square is given. Write and solve an equation to find the length of each side of the square.**

48. $P = 36$ in. **$4s = 36$; 9 in.**
49. $P = 84$ in. **$4s = 84$; 21 in.**
50. $P = 100$ yd **$4s = 100$; 25 yd**
51. $P = 16.4$ cm **$4s = 16.4$; 4.1 cm**

Write an equation to represent each relationship. Then solve the equation.

52. Five times a number is 45. **$5x = 45$; $x = 9$**
53. A number multiplied by negative 3 is 12. **$-3x = 12$; $x = -4$**
54. A number divided by 4 is equal to 10. **$\frac{x}{4} = 10$; $x = 40$**
55. The quotient of a number and 3 is negative 8. **$\frac{x}{3} = -8$; $x = -24$**

56. **Statistics** The mean height of the students in Marta's class is 60 in. There are 18 students in her class. Write and solve an equation to find the total measure of all students' heights. (*Hint:* The mean is found by dividing the sum of all data values by the number of data values.) **$\frac{x}{18} = 60$; 1080**

57. **Finance** Lisa earned \$6.25 per hour at her after-school job. Each week she earned \$50. Write and solve an equation to show how many hours she worked each week. **$6.25h = 50$; 8 h**

58. **Critical Thinking** Will the solution of $\frac{x}{2.1} = 4$ be greater than 4 or less than 4? Explain. **Greater than 4; you are multiplying 4 by a positive number greater than 1.**

59. **Consumer Economics** Dion's long-distance phone bill was \$13.80. His long-distance calls cost \$0.05 per minute. Write and solve an equation to find the number of minutes he was charged for. Show that your answer is reasonable. **$0.05m = 13.80$; 276 min**

60. **Nutrition** An 8 oz cup of coffee has about 184 mg of caffeine. This is 5 times as much caffeine as in a 12 oz soft drink. Write and solve an equation to find about how much caffeine is in a 12 oz caffeinated soft drink. Round your answer to the nearest whole number. Show that your answer is reasonable. **$5c = 184$; 37 mg**

LINK Statistics

American Robert P. Wadlow (1918–1940) holds the record for world's tallest man—8 ft 11.1 in. He also holds world records for the largest feet and hands.

Source: Guinness World Records 2005

Use the equation $8y = 4x$ to find y for each value of x.

	x	$4x$	$8y = 4x$	y
61.	−4	$4(-4) = -16$	$8y = -16$	**−2**
62.	−2	**−8**	**$8y = -8$**	**−1**
63.	0	**0**	**$8y = 0$**	**0**
64.	2	**8**	**$8y = 8$**	**1**

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PRACTICE A

PRACTICE C

PRACTICE B

Solve each equation. Check your answers.

1. $\frac{d}{8} = 6$ **$d = 48$**
2. $-5 = \frac{n}{2}$ **$n = -10$**
3. $2p = 54$ **$p = 27$**
4. $\frac{-t}{2} = 12$ **$t = -24$**
5. $-40 = -4x$ **$x = 10$**
6. $\frac{2r}{3} = 16$ **$r = 24$**
7. $-49 = 7y$ **$y = -7$**
8. $-15 = -\frac{3n}{5}$ **$n = 25$**
9. $9m = 6$ **$m = \frac{2}{3}$**
10. $\frac{v}{-3} = -6$ **$v = 18$**
11. $2.8 = \frac{b}{4}$ **$b = 11.2$**
12. $\frac{3r}{4} = \frac{1}{8}$ **$r = \frac{1}{6}$**

Answer each of the following.

13. The perimeter of a regular pentagon is 41.5 cm. Write and solve an equation to determine the length of each side of the pentagon. **$5x = 41.5$; 8.3 cm**
14. In June 2005, Peter mailed a package from his local post office in Fayetteville, North Carolina to a friend in Radford, Virginia for \$2.07. The first-class rate at the time was \$0.23 per ounce. Write and solve an equation to determine the weight of the package. **$0.23x = 2.07$; 9 oz**
15. Lola spends one-third of her allowance on movies. She spends \$8 per week at the movies. Write and solve an equation to determine Lola's weekly allowance. **$\frac{1}{3}x = 8$; \$24**

READING STRATEGIES

When solving multiplication and division equations, people often make mistakes that can easily be avoided by making a prediction first. Look at the examples below to see how you can make predictions.

EX. $\frac{n}{8} = 2$ — Think, "Some number divided by 8 equals 2." → When dividing with whole numbers, the dividend is the greatest value.

Prediction: n is greater than both 8 and 2.
If your solution is $n = 4$, you know it's incorrect. Here, $n = 16$.

EX. $8 = 2n$ — Think, "2 times some number equals 8." → When multiplying with whole numbers, the factors are less than the product.

Prediction: n is less than 8.
If your answer is $n = 16$, you know it's incorrect. Here, $n = 4$.

EX. $\frac{1}{2}n = 8$ — Think, "Half of some number equals 8." → When multiplying by a fraction, the missing factor is greater than the product.

Prediction: n is greater than 8.
If your solution is $n = 4$, you know it's incorrect. Here, $n = 16$.

Make predictions by filling in each blank with "less" or "greater".

1. For $6t = 12$, t must be **less** than 12.
2. For $\frac{1}{4}f = 16$, f must be **greater** than 16.
3. For $5 = \frac{y}{10}$, y must be **greater** than 5.

Make a mental prediction about each solution. Then solve.

4. $3b = 12$ **$b = 4$**
5. $\frac{1}{3}c = 6$ **$c = 18$**
6. $10 = \frac{k}{2}$ **$k = 20$**

RETEACH

Solve equations involving multiplication and division by performing the inverse operation.

Solve $\frac{x}{5} = 4$. **Check:** $\frac{x}{5} = 4$

$\frac{x}{5} = 4$ — x is divided by 5. — $\frac{20}{5} \stackrel{?}{=} 4$

$5 \cdot \frac{x}{5} = 4 \cdot 5$ — Multiply both sides by 5. — $4 \stackrel{?}{=} 4$ ✓

$\frac{5x}{5} = 20$ — Simplify.

$x = 20$

Solve $-3x = 27$. **Check:** $-3x = 27$

$-3x = 27$ — x is multiplied by −3. — $-3(-9) \stackrel{?}{=} 27$

$\frac{-3x}{-3} = \frac{27}{-3}$ — Divide both sides by −3. — $27 \stackrel{?}{=} 27$ ✓

$x = -9$ — Simplify.

Circle the correct word in each sentence. Then solve the equation.

1. $\frac{x}{-2} = 7$
   x is multiplied/divided by −2.
   To solve, multiply/divide both sides by −2.
   $x = -14$
2. $5m = -40$
   m is multiplied/divided by 5.
   To solve, multiply/divide both sides by 5.
   $m = -8$

Solve each equation. Check your answers.

3. $-2x = -20$ **10**
4. $\frac{w}{5} = -7$ **−35**
5. $6z = -42$ **−7**

MULTI-STEP TEST PREP

65. a. The formula for the mean of a data set is mean $= \frac{\text{sum of data values}}{\text{number of data values}}$. One summer, there were 1926 wildfires in Arizona. Which value does this number represent in the formula? **number of data values**

b. The mean number of acres burned by each wildfire was 96.21. Which value does this number represent in the formula? **mean**

c. Use the formula and information given to find how many acres were burned by wildfires in Arizona that summer. Round your answer to the nearest acre. Show that your answer is reasonable. **185,300 acres**

Solve each equation. Check your answer.

66. $\frac{m}{6} = 1$ **6**

67. $4x = 28$ **7**

68. $1.2h = 14.4$ **12**

69. $\frac{1}{5}x = 121$ **605**

70. $2w = 26$ **13**

71. $4b = \frac{3}{4}$ $\frac{3}{16}$

72. $5y = 11$ $\frac{11}{5}$, or 2.2

73. $\frac{n}{1.9} = 3$ **5.7**

Biology **Use the table for Exercises 74 and 75.**

Average Weight			
Animal	**At Birth (g)**	**Adult Female (g)**	**Adult Male (g)**
Hamster	2	130	110
Guinea pig	85	800	1050
Rat	5	275	480

74. The mean weight of an adult male rat is 16 times the mean weight of an adult male mouse. Write and solve an equation to find the mean weight of an adult male mouse. Show that your answer is reasonable. **$16m = 480$; 30 g**

75. On average, a hamster at birth weighs $\frac{2}{3}$ the weight of a gerbil at birth. Write and solve an equation to find the average weight of a gerbil at birth. Show that your answer is reasonable.

75. $\frac{2}{3}g = 2$; 3 g

76. Write About It Describe a real-world situation that can be modeled by $3x = 42$. Solve the equation and tell what the solution means in the context of your problem.

76. Possible answer: Ryan has 3 times as many baseball cards as David. Ryan has 42 cards. How many cards does David have? $x = 14$; this represents the number of cards that David has.

TEST PREP

77. Which situation does NOT represent the equation $\frac{d}{2} = 10$?

Ⓐ Leo bought a box of pencils. He gave half of them to his brother. They each got 10 pencils. How many pencils were in the box Leo bought?

Ⓑ Kasey evenly divided her money from baby-sitting into two bank accounts. She put $10 in each account. How much did Kasey earn?

Ⓒ Gilbert cut a piece of ribbon into 2-inch strips. When he was done, he had ten 2-inch strips. How long was the ribbon to start?

Ⓓ Mattie had 2 more CDs than her sister Leona. If Leona had 10 CDs, how many CDs did Mattie have?

78. Which equation below shows a correct first step for solving $3x = -12$?

Ⓕ $3x + 3 = -12 + 3$

Ⓖ $3x - 3 = -12 - 3$

Ⓗ $3(3x) = 3(-12)$

Ⓙ $\frac{3x}{3} = \frac{-12}{3}$

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MULTI-STEP TEST PREP **Exercise 65** involves using the formula for the mean of a data set to find the total acreage burned by wildfires. This exercise prepares students for the Multi-Step Test Prep.

Answers

65c. Possible answer: there were about 1900 fires and the mean acreage burned was about 100. So the total acres burned will be close to $1900(100) = 190{,}000$. So 185,300 is reasonable.

74. Possible answer: an adult male rat weighs about 500 g, and this is about 20 times the weight of an adult mouse. So the mouse will weigh close to $\frac{500}{20} = 25$ g. So 30 g is reasonable.

75. Possible answer: a hamster weighs 2 g, so the gerbil's weight will be greater than 2 g. So 3 g is reasonable.

PROBLEM SOLVING

Write the correct answer.

1. John threw a surprise birthday party for his friend. Food, drinks, and a DJ cost $480 for a group of 32 people. Write and solve an equation to find the cost c per person.
   $32c = 480$; $15

2. One serving of soybeans contains 10 grams of protein, which is 4 times the amount in one serving of kale. Write and solve an equation to find the amount of protein x in one serving of kale.
   $4x = 10$; 2.5 grams

3. Maria earned $10.50 per hour working at an ice cream shop. She earned $147 each week before taxes. Write and solve an equation to find the number of hours h she worked each week.
   $10.50h = 147$; 14 hours

4. Ben is saving $\frac{1}{5}$ of his weekly pay to buy a car. Write and solve an equation to find what weekly pay w results in savings of $61.50.
   $\frac{1}{5}w = 61.50$; $307.50

Use the table below to answer questions 5–7. Select the best answer.
The table shows the maximum speed in miles per hour for various animals.

Animal	mi/h
Falcon	200
Zebra	40
Cat (domestic)	30
Black Mamba Snake	20
Snail	0.03

5. The speed of a snail is how many times that of a cat?
   A $\frac{1}{1000}$ C 100
   B $\frac{1}{100}$ D 1000

6. A cheetah's maximum speed of 70 mi/h is x times faster than a black mamba snake's maximum speed. Which equation shows this relationship?
   F $20 + x = 70$ H $70 = \frac{20}{x}$
   G $20 = 70x$ J $70 = 20x$

7. Use your equation in problem 6 to find how many times faster a cheetah is than a black mamba snake if they are both traveling at their maximum speed.
   A 0.3 times C 10 times
   B 3.5 times D 50 times

CHALLENGE

The Multiplication Property of Equality states that if a, b, and c are numbers and $a = b$, then $ac = bc$. That is, when you multiply each side of an equation by a fixed number, the resulting products are equal.

By itself, you can use the Multiplication Property of Equality to solve equations of the form $ax = b$ as well as equations of the form $\frac{x}{a} = b$.

You can see this fact at work in the solution below.

$3x = 6$

$\frac{1}{3}(3x) = \frac{1}{3}(6)$

$\left[\frac{1}{3}(3)\right]x = \frac{1}{3}(6)$

$1x = \frac{1}{3}(6)$

$x = 2$

Notice that you multiply by a fraction whose numerator is 1, that is, by the reciprocal of a.

Solve each equation by using only the Multiplication Property of Equality. Show your work.

1. $4x = 12$ **$x = 3$**
2. $-3x = 15$ **$x = -5$**
3. $-7x = -21$ **$x = 3$**
4. $10t = 12$ **$t = \frac{6}{5}$**
5. $18w = 9$ **$w = \frac{1}{2}$**
6. $-3d = -90$ **$d = 30$**

7. Suppose that $a \neq 0$. Using the process that you followed in Exercises 1–6, solve $ax = b$ for x.
   $ax = b$; $\frac{1}{a} \cdot ax = \frac{1}{a} \cdot b$; $1 \cdot x = \frac{1}{a} \cdot b$; $x = \frac{b}{a}$

8. Use the Division Property of Equality to solve $ax = b$ for x.
   $\frac{ax}{a} = \frac{b}{a}$; $1x = \frac{b}{a}$; $x = \frac{b}{a}$

9. Compare the method of solving an equation of the form $ax = b$ by using the Multiplication Property of Equality with that of solving the same type of equation by using the Division Property of Equality.
   Using the Multiplication Property of Equality, multiply each side of the equation by the reciprocal of the coefficient of the x-term. Using the Division Property of Equality, divide each side of the equation by the coefficient of the x-term. By either method, the solution is the same.

TEST PREP DOCTOR In **Exercise 79,** students who chose **D** probably understand that the sum is being divided by 5, but may not understand that the unknown is each angle measure. Point out that the sum is 540, so if written as a division, the equation would be $\frac{540}{5} = x$. Remind students that a sum is that same as repeated multiplication, so the sum of 5 equal angles can be expressed as $5x$, which leads to choosing **B.**

Journal

The equation $\frac{3}{4}x = 7.2$ could be solved using either all fractions or all decimals. Have students explain which way they would solve it and why, and then have them solve it.

ALTERNATIVE ASSESSMENT

Have students make a two-column chart. On the left side, tell them to write and solve three equations by using the Multiplication Property of Equality. On the right side, tell them to write and solve three equations by using the Division Property of Equality. Then have them write a real-world scenario for one equation from each side.

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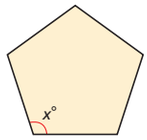
1-3 Lesson Quiz

Solve each equation.

1. $-3 = \frac{m}{-7}$ 21
2. $\frac{x}{100} = 0.028$ 2.8
3. $8y = 4$ $\frac{1}{2}$
4. $126 = -9q$ -14
5. $\frac{2}{5}m = 16$ 40
6. $\frac{15}{16}c = \frac{25}{48}$ $\frac{5}{9}$
7. A person's weight on Venus is about $\frac{9}{10}$ his or her weight on Earth. Write and solve an equation to find how much a person weighs on Earth if he or she weighs 108 pounds on Venus. $108 = \frac{9}{10}w$; 120 lb

Also available online

79. In a regular pentagon, all of the angles are equal in measure. The sum of the angle measures is 540°. Which of the following equations could be used to find the measure of each angle?



Ⓐ $\frac{x}{540} = 5$ Ⓒ $540x = 5$

Ⓑ $5x = 540$ Ⓓ $\frac{x}{5} = 540$

80. For which equation is $m = 10$ a solution?

Ⓕ $5 = 2m$ Ⓖ $5m = 2$ Ⓗ $\frac{m}{2} = 5$ Ⓙ $\frac{m}{10} = 2$

81. Short Response Luisa bought 6 cans of cat food that each cost the same amount. She spent a total of $4.80.

a. Write an equation to determine the cost of one can of cat food. Tell what each part of your equation represents.

b. Solve your equation to find the cost of one can of cat food. Show each step.

81a. $6c = 4.80$; 6 is the number of cans of cat food; c is the cost per can; 4.80 is the total cost.

b. $c = \$0.80$

CHALLENGE AND EXTEND

Solve each equation. Check your answer.

82. $\left(3\frac{1}{5}\right)b = \frac{4}{5}$ $\frac{1}{4}$

83. $\left(1\frac{1}{3}\right)x = 2\frac{2}{3}$ 2

84. $\left(5\frac{4}{5}\right)x = -52\frac{1}{5}$ -9

85. $\left(-2\frac{9}{10}\right)k = -26\frac{1}{10}$ 9

86. $\left(1\frac{2}{3}\right)w = 15\frac{1}{3}$ $\frac{46}{5}$

87. $\left(2\frac{1}{4}\right)d = 4\frac{1}{2}$ 2

Find each indicated value.

88. If $2p = 4$, find the value of $6p + 10$. 22

89. If $6t = 24$, find the value of $-5t$. -20

90. If $3x = 15$, find the value of $12 - 4x$. -8

91. If $\frac{n}{2} = -11$, find the value of $6n$. -132

92. To isolate x in $ax = b$, what should you divide both sides by? a

93. To isolate x in $\frac{x}{a} = b$, what operation should you perform on both sides of the equation? Multiply both sides by a.

94. Travel The formula $d = rt$ gives the distance d that is traveled at a rate r in time t.

a. If $d = 400$ and $r = 25$, what is the value of t? 16

b. If $d = 400$ and $r = 50$, what is the value of t? 8

c. What if...? How did t change when r increased from 25 to 50? t is divided in half (from 16 to 8).

d. What if...? If r is doubled while d remains the same, what is the effect on t? t is divided in half.

1-4 Technology LAB

Solve Equations by Graphing

You can use graphs to solve equations. As you complete this activity, you will learn some of the connections between graphs and equations.

Use with Solving Two-Step and Multi-Step Equations

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Learn It Online Lab Resources Online

Activity

Solve $3x - 4 = 5$.

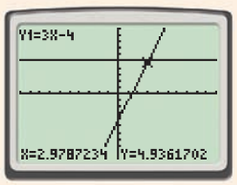
1. Press Y=. In Y_1, enter the left side of the equation, $3x - 4$.

   Y= 3 X,T,θ,n − 4 ENTER

   In Y_2, enter the right side of the equation, 5.

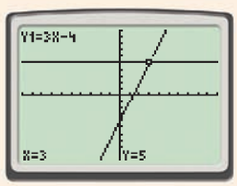
   Y= 5 ENTER

2. Press GRAPH. Press TRACE. The display will show the x- and y-values of a point on the first line. Press the right arrow key several times. Notice that the x- and y-values change.



3. Continue to trace as close as possible to the intersection of the two lines. The x-value of this point 2.9787…, is an approximation of the solution. The solution is about 3.

4. While still in trace mode, to check, press 3 ENTER. The display will show the y-value when $x = 3$. When $x = 3$, $y = 5$. So 3 is the solution. You can also check this solution by substituting 3 for x in the equation:



Check

$3x - 4 = 5$	
$3(3) - 4$	5
$9 - 4$	5
5	5 ✓

Try This

1. Solve $3x - 4 = 2$, $3x - 4 = 17$, and $3x - 4 = -7$ by graphing. **2; 7; −1**
2. What does each line represent? **Each line represents one side of the equation.**
3. Describe a procedure for finding the solution of $3x - 4 = y$ for any value of y.
4. Solve $\frac{1}{2}x - 7 = -4$, $\frac{1}{2}x - 7 = 0$, and $\frac{1}{2}x - 7 = 2$ by graphing. **6; 14; 18**

Teaching Tip **Technology** Students should set the window to the standard view before the activity. Press ZOOM and select **6:ZStandard**.

Technology LAB Organizer

Use with Solving Two-Step and Multi-Step Equations

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use a graphing calculator to solve two-step equations by graphing.

Materials: graphing calculator

Online Edition Graphing Calculator, TechKeys

Countdown

Teach

Discuss

Have students use the Table feature to make a table of values for $y = 3x - 4$. Find x when $y = 5$. Discuss how this solution method relates to solving by graphing.

Close

Key Concept

For an equation in the form *expression* = *constant*, graph $y =$ *expression* and $y =$ *constant*. The solution is the x-coordinate of the point where the two graphs intersect.

Assessment

Journal Have students explain how to solve an equation by graphing. They should include an example in their explanation.

Answers

3. Graph $y = 3x - 4$ and $y =$ *given value*. Find the intersection point of the two lines. The intersection point's x-coordinate is the solution.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Learn It Online State Resources Online

1-4 Organizer

Pacing: Traditional $1\frac{1}{2}$ days
Block $\frac{3}{4}$ day

Objective: Solve equations in one variable that contain more than one operation.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Evaluate each expression.

1. $9 - 3(-2)$ 15
2. $3(-5 + 7)$ 6
3. $12\left(\frac{3 + (-7)}{12}\right)$ -4
4. $26 - 4(7 - 5)$ 18

Simplify each expression.

5. $10c + c$ $11c$
6. $8.2b + 3.8b - 12b$ 0
7. $5m + 2(2m - 7)$ $9m - 14$
8. $6x - (2x + 5)$ $4x - 5$

Also available online

Math Humor

Q: How do equations get into shape?

A: They do multi-step aerobics.

COMMON CORE

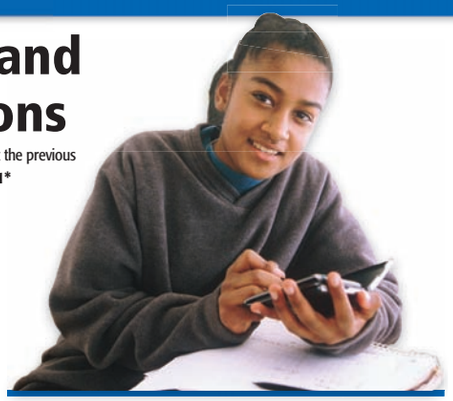
CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution.... *Also* **CC.9-12.A.REI.3**, **CC.9-12.A.CED.1***

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *Also* **CC.9-12.A.CED.1***

Learn It Online
State Resources Online

COMMON CORE

1-4 Solving Two-Step and Multi-Step Equations



CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution.... *Also* **CC.9-12.A.REI.3**, **CC.9-12.A.CED.1***

Objective
Solve equations in one variable that contain more than one operation.

Why learn this?
Equations containing more than one operation can model real-world situations, such as the cost of a music club membership.

Alex belongs to a music club. In this club, students can buy a student discount card for $19.95. This card allows them to buy CDs for $3.95 each. After one year, Alex has spent $63.40.

To find the number of CDs c that Alex bought, you can solve an equation.

Cost of discount card ↓

Cost per CD → $3.95c + 19.95 = 63.40$ ← Total cost

Notice that this equation contains multiplication and addition. Equations that contain more than one operation require more than one step to solve. Identify the operations in the equation and the order in which they are applied to the variable. Then use inverse operations and work backward to undo them one at a time.

$$3.95c + 19.95 = 63.40$$

Operations in the Equation		To Solve
1 First c is multiplied by 3.95.	Work Backward	1 Subtract 19.95 from both sides of the equation.
2 Then 19.95 is added.		2 Then divide both sides by 3.95.

EXAMPLE 1 **Solving Two-Step Equations**

Solve $10 = 6 - 2x$. Check your answer.

$10 = 6 - 2x$ *First x is multiplied by −2. Then 6 is added.*
$-6 \quad -6$ *Work backward: Subtract 6 from both sides.*
$4 = -2x$
$\frac{4}{-2} = \frac{-2x}{-2}$ *Since x is multiplied by −2, divide both sides by −2 to undo the multiplication.*
$-2 = 1x$
$-2 = x$

Check

$10 = 6 - 2x$	
10	$6 - 2(-2)$
10	$6 - (-4)$
10	10 ✓

CHECK IT OUT! **Solve each equation. Check your answer.**

1a. $-4 + 7x = 3$ 1 **1b.** $1.5 = 1.2y - 5.7$ 6 **1c.** $\frac{n}{7} + 2 = 2$ 0

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1 Introduce

EXPLORATION

Solving Two-Step and Multi-Step Equations

Juan wants to rent a bike while on vacation. The sign posted on the rental stand says "$10 fee plus $3 per hour."

1. Juan writes the expression $10 + 3h$ to find the cost of renting a bike for h hours. Complete the table.

Hours h	Cost $10 + 3h$
1	13
2	16
3	19
4	22
5	25

2. Find the hours that correspond to the costs in the table.

Hours h	Cost $10 + 3h$
6	28
8	34
10	40
11	43
12	46

THINK AND DISCUSS

3. **Explain** how you evaluated $10 + 3h$ for a given value of h. Multiply the value of h by 3 and then add 10.
4. **Describe** how you completed the table in Problem 2. What process did you use? Subtract 10 from the cost and then divide by 3.

Motivate

Tell students that in the morning, you get in your car, start it, drive to school, turn off the car, and get out. To undo this chain of events, you get in the car, start it, drive home, turn off the car, and get out. Stress that your "undoing" must be exactly in reverse order; otherwise you might get out of the car while it is still running.

Solving equations with more than one operation is similar. The variable is isolated by reversing the order of operations that were used on it.

Explorations and answers are provided in your online resources.

EXAMPLE 2 Solving Two-Step Equations That Contain Fractions

Solve $\frac{q}{15} - \frac{1}{5} = \frac{3}{5}$.

Method 1 Use fraction operations.

$\frac{q}{15} - \frac{1}{5} = \frac{3}{5}$ — *Since $\frac{1}{5}$ is subtracted from $\frac{q}{15}$, add $\frac{1}{5}$ to both sides to undo the subtraction.*

$+\frac{1}{5} \quad +\frac{1}{5}$

$\frac{q}{15} = \frac{4}{5}$ — *Since q is divided by 15, multiply both sides by 15 to undo the division.*

$15\left(\frac{q}{15}\right) = 15\left(\frac{4}{5}\right)$

$q = \frac{15 \cdot 4}{5}$ — *Simplify.*

$q = \frac{60}{5}$

$q = 12$

Method 2 Multiply by the least common denominator (LCD) to clear the fractions.

$\frac{q}{15} - \frac{1}{5} = \frac{3}{5}$

$15\left(\frac{q}{15} - \frac{1}{5}\right) = 15\left(\frac{3}{5}\right)$ — *Multiply both sides by 15, the LCD of the fractions.*

$15\left(\frac{q}{15}\right) - 15\left(\frac{1}{5}\right) = 15\left(\frac{3}{5}\right)$ — *Distribute 15 on the left side.*

$q - 3 = 9$ — *Simplify.*

$+3 \quad +3$

$q = 12$ — *Since 3 is subtracted from q, add 3 to both sides to undo the subtraction.*

CHECK IT OUT! Solve each equation. Check your answer.

2a. $\frac{2x}{5} - \frac{1}{2} = 5$ $\frac{55}{4}$ **2b.** $\frac{3}{4}u + \frac{1}{2} = \frac{7}{8}$ $\frac{1}{2}$ **2c.** $\frac{1}{5}n - \frac{1}{3} = \frac{8}{3}$ 15

Equations that are more complicated may have to be simplified before they can be solved. You may have to use the Distributive Property or combine like terms before you begin using inverse operations.

EXAMPLE 3 Simplifying Before Solving Equations

Solve each equation.

A $6x + 3 - 8x = 13$

$6x + 3 - 8x = 13$

$6x - 8x + 3 = 13$ — *Use the Commutative Property of Addition.*

$-2x + 3 = 13$ — *Combine like terms.*

$-3 \quad -3$

$-2x = 10$ — *Since 3 is added to −2x, subtract 3 from both sides to undo the addition.*

$\frac{-2x}{-2} = \frac{10}{-2}$ — *Since x is multiplied by −2, divide both sides by −2 to undo the multiplication.*

$x = -5$

COMMON ERROR ALERT

When clearing fractions in an equation, students may forget to multiply every term by the LCD, especially when one of the terms is not a fraction. For instance, in **Example 2,** they may write $15 \cdot \frac{q}{15} - \frac{1}{5} = 15 \cdot \frac{3}{5}$, and then fail to multiply 15 by $-\frac{1}{5}$ on the left side. Remind them to use parentheses to show multiplication by the LCD and to distribute the LCD to each term.

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Additional Examples

Example 1

Solve each equation.

A. $18 = 4a + 10$ 2

B. $5t - 2 = -32$ −6

Example 2

Solve each equation.

A. $\frac{y}{8} - \frac{3}{4} = \frac{5}{4}$ 16

B. $\frac{2}{3}r + \frac{3}{4} = \frac{7}{12}$ $-\frac{1}{4}$

Example 3

Solve each equation.

A. $8x - 21 - 5x = -15$ 2

B. $10y - (4y + 8) = -20$ −2

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- What steps need to be taken to isolate the variable? Why?

EXAMPLE 2

- What happens to the fractions in an equation when both sides are multiplied by the LCD?

EXAMPLE 3

- How do you know what signs go with each term in the equation?

2 Teach

Guided Instruction

Remind students that the goal in solving equations is to isolate the variable. Emphasize using the Distributive Property as needed before combining like terms, and make special note of distributing a negative number. Encourage students to work carefully and do all their work on paper. Trying to solve multi-step equations mentally often results in careless errors.

Reaching All Learners *Through Critical Thinking*

Have students create their own multi-step equation by writing a simple equation and performing operations, one at a time, on both sides. For example:

$x = 3$

$x + 2 = 5$ Add 2.

$4(x + 2) = 20$ Multiply by 4.

Then have students solve their own multi-step equation. They should get back to their original equation.

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Additional Examples

Example 4

Jan joined the dining club at the local café for a fee of $29.95. Being a member entitles her to save $2.50 every time she buys lunch. So far, Jan calculates that she has saved a total of $12.55 by joining the club. Write and solve an equation to find how many times Jan has eaten lunch at the café. $12.55 = 2.50c - 29.95$; $c = 17$

Also available online

Questioning Strategies

EXAMPLE 4

- How do you decide what the variable will represent?
- What operations are needed to solve the equation?

Solve each equation.

B $9 = 6 - (x + 2)$

$9 = 6 + (-1)(x + 2)$ *Write subtraction as addition of the opposite.*

$9 = 6 + (-1)(x) + (-1)(2)$ *Distribute −1 on the right side.*

$9 = 6 - x - 2$ *Simplify.*

$9 = 6 - 2 - x$ *Use the Commutative Property of Addition.*

$9 = 4 - x$ *Combine like terms.*

$-4 \quad -4$ *Since 4 is added to −x, subtract 4 from both sides to undo the addition.*

$5 = -x$

$\frac{5}{-1} = \frac{-x}{-1}$ *Since x is multiplied by −1, divide both sides by −1 to undo the multiplication.*

$-5 = x$

Helpful Hint

You can think of an opposite sign as a coefficient of −1. $-(x + 2) = -1(x + 2)$ and $-x = -1x$.

CHECK IT OUT! **Solve each equation. Check your answer.**

3a. $2a + 3 - 8a = 8$ $-\frac{5}{6}$

3b. $-2(3 - d) = 4$ 5

3c. $4(x - 2) + 2x = 40$ 8

EXAMPLE 4 ***Problem-Solving Application***

Alex belongs to a music club. In this club, students can buy a student discount card for $19.95. This card allows them to buy CDs for $3.95 each. After one year, Alex has spent $63.40. Write and solve an equation to find how many CDs Alex bought during the year.

Make sense of problems and persevere in solving them.

1 Understand the Problem

The **answer** will be the number of CDs that Alex bought during the year.

List the important information:

- Alex paid $19.95 for a student discount card.
- Alex pays $3.95 for each CD purchased.
- After one year, Alex has spent $63.40.

2 Make a Plan

Let c represent the number of CDs that Alex purchased. That means Alex has spent $3.95c. However, Alex must also add the amount spent on the card. Write an equation to represent this situation.

total cost	=	cost of compact discs	+	cost of discount card
63.40	=	3.95c	+	19.95

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3 Solve

$$63.40 = 3.95c + 19.95$$
$$-19.95 \quad -19.95$$
$$43.45 = 3.95c$$
$$\frac{43.45}{3.95} = \frac{3.95c}{3.95}$$
$$11 = c$$

Since 19.95 is added to 3.95c, subtract 19.95 from both sides to undo the addition.

Since c is multiplied by 3.95, divide both sides by 3.95 to undo the multiplication.

Alex bought 11 CDs during the year.

4 Look Back

Check that the answer is reasonable. The cost per CD is about \$4, so if Alex bought 11 CDs, this amount is about $11(4) = \$44$.

Add the cost of the discount card, which is about \$20: $44 + 20 = 64$. So the total cost was about \$64, which is close to the amount given in the problem, \$63.40.

CHECK IT OUT! **4.** Sara paid \$15.95 to become a member at a gym. She then paid a monthly membership fee. Her total cost for 12 months was \$735.95. How much was the monthly fee? **\$60**

EXAMPLE 5 Solving Equations to Find an Indicated Value

If $3a + 12 = 30$, find the value of $a + 4$.

Step 1 Find the value of a.

$$3a + 12 = 30$$
$$-12 \quad -12$$
$$3a = 18$$
$$\frac{3a}{3} = \frac{18}{3}$$
$$a = 6$$

Since 12 is added to 3a, subtract 12 from both sides to undo the addition.

Since a is multiplied by 3, divide both sides by 3 to undo the multiplication.

Step 2 Find the value of $a + 4$.

$$a + 4$$
$$6 + 4$$
$$10$$

To find the value of a + 4, substitute 6 for a.

Simplify.

CHECK IT OUT! **5.** If $2x + 4 = -24$, find the value of $3x$. **-42**

THINK AND DISCUSS

1. Explain the steps you would follow to solve $2x + 1 = 7$. How is this procedure different from the one you would follow to solve $2x - 1 = 7$?

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write and solve a multi-step equation. Use addition, subtraction, multiplication, and division at least one time each.

Solving Multi-Step Equations	

COMMON ERROR ALERT

When solving equations, students sometimes write only numbers, not numbers with their operation, on each side. This can lead to careless errors. For example:

$$3x + 12 = 45$$
$$\frac{12}{3x} \quad \frac{12}{= 57}$$

Emphasize to students the importance of writing every part of each step.

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Additional Examples

Example 5

A. If $4a + 0.2 = 5$, find the value of $a - 1$. 0.2

B. If $3d - (9 - 2d) = 51$, find the value of $3d$. 36

Also available online

Questioning Strategies

EXAMPLE 5

- What are the two main steps in solving these types of problems?

3 Close

Summarize

Ask students what they would do as a first step in solving each of the following:

- $7x - 3 = 29$ Add 3 to both sides.
- $5y - 2(y - 6) = 43$ Distribute -2.
- $-\frac{1}{3}x + \frac{3}{4} = \frac{7}{12}$ Multiply both sides by 12, or subtract $\frac{3}{4}$ from both sides.
- $25 = -m - 9 + 3m$ Combine like terms.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. Possible answer: To solve $2x + 1 = 7$, first subtract 1 from both sides, and then divide both sides by 2. To solve $2x - 1 = 7$, first add 1 to both sides, then divide by 2.
2. See Additional Answers.

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–3**

- **Basic** 24–41, 47–51, 54–61
- **Average** 24–41, 47–51, 54–61, 74, 75
- **Advanced** 24–41, 48, 54–61, 74–77

If you finished Examples **1–5**

- **Basic** 24–63, 67, 69–73
- **Average** 24–51, 53–61 even, 62, 63, 65, 67–80
- **Advanced** 24–46, 48, 50, 55–61 even, 62–80

Homework Quick Check

Quickly check key concepts.
Exercises: 24, 32, 40, 42, 44, 48, 50

Answers

47. $2x + 100 = 180$
48. $2x + 115 = 180$
49. $4x + 40 = 180$

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 19, 42, 53, 62–67, 73

Construct viable arguments and critique the reasoning of others. Exercises 68–69

Model with mathematics. Exercises 47–49

Learn It Online
State Resources Online

1-4 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

Solve each equation. Check your answer.

SEE EXAMPLE 1

1. $4a + 3 = 11$ 2
2. $8 = 3r - 1$ 3
3. $42 = -2d + 6$ −18
4. $x + 0.3 = 3.3$ 3
5. $15y + 31 = 61$ 2
6. $9 - c = -13$ 22

SEE EXAMPLE 2

7. $\frac{x}{6} + 4 = 15$ 66
8. $\frac{1}{3}y + \frac{1}{4} = \frac{5}{12}$ $\frac{1}{2}$
9. $\frac{2}{7}j - \frac{1}{7} = \frac{3}{14}$ $\frac{5}{4}$
10. $15 = \frac{a}{3} - 2$ 51
11. $4 - \frac{m}{2} = 10$ −12
12. $\frac{x}{8} - \frac{1}{2} = 6$ 52

SEE EXAMPLE 3

13. $28 = 8x + 12 - 7x$ 16
14. $2y - 7 + 5y = 0$ 1
15. $2.4 = 3(m + 4)$ −3.2
16. $3(x - 4) = 48$ 20
17. $4t + 7 - t = 19$ 4
18. $5(1 - 2w) + 8w = 15$ −5

SEE EXAMPLE 4

19. **Transportation** Paul bought a student discount card for the bus. The card cost $7 and allows him to buy daily bus passes for $1.50. After one month, Paul spent $29.50. How many daily bus passes did Paul buy? 15 passes

SEE EXAMPLE 5

20. If $3x - 13 = 8$, find the value of $x - 4$. 3
21. If $3(x + 1) = 7$, find the value of $3x$. 4
22. If $-3(y - 1) = 9$, find the value of $\frac{1}{2}y$. −1
23. If $4 - 7x = 39$, find the value of $x + 1$. −4

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
24–29	1
30–35	2
36–41	3
42	4
43–46	5

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

Solve each equation. Check your answer.

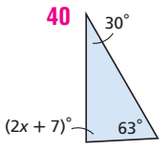
24. $5 = 2g + 1$ 2
25. $6h - 7 = 17$ 4
26. $0.6v + 2.1 = 4.5$ 4
27. $3x + 3 = 18$ 5
28. $0.6g + 11 = 5$ −10
29. $32 = 5 - 3t$ −9
30. $2d + \frac{1}{5} = \frac{3}{5}$ $\frac{1}{5}$
31. $1 = 2x + \frac{1}{2}$ $\frac{1}{4}$
32. $\frac{z}{2} + 1 = \frac{3}{2}$ 1
33. $\frac{2}{3} = \frac{4j}{6}$ 1
34. $\frac{3}{4} = \frac{3}{8}x - \frac{3}{2}$ 6
35. $\frac{1}{5} - \frac{x}{5} = -\frac{2}{5}$ 3
36. $6 = -2(7 - c)$ 10
37. $5(h - 4) = 8$ $\frac{28}{5}$
38. $-3x - 8 + 4x = 17$ 25
39. $4x + 6x = 30$ 3
40. $2(x + 3) = 10$ 2

42. **Consumer Economics** Jennifer is saving money to buy a bike. The bike costs $245. She has $125 saved, and each week she adds $15 to her savings. How long will it take her to save enough money to buy the bike? 8 weeks

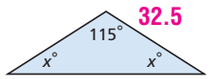
43. If $2x + 13 = 17$, find the value of $3x + 1$. 7
44. If $-(x - 1) = 5$, find the value of $-4x$. 16
45. If $5(y + 10) = 40$, find the value of $\frac{1}{4}y$. $-\frac{1}{2}$
46. If $9 - 6x = 45$, find the value of $x - 4$. −10

Geometry Write and solve an equation to find the value of x for each triangle. (*Hint:* The sum of the angle measures in any triangle is 180°.)

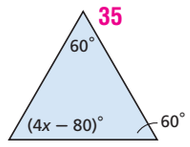
47. 40



48. 32.5



49. 35



READING STRATEGIES

Use the example below to understand the procedure for solving multi-step equations. Not all problems will require all the steps.

Solve $-2(x - 5) + 4x = 17$.

$-2(x - 5) + 4x = 17$ → 1. Use the Distributive Property.
$-2x + 10 + 4x = 17$
$-2x + 10 + 4x = 17$ → 2. Identify and combine like terms.
$2x + 10 = 17$
$2x + 10 = 17$
$-10 \quad -10$ → 3. "Undo" addition (and subtraction).
$\frac{2x}{2} = \frac{7}{2}$ → 4. "Undo" division (and multiplication).
$x = 3.5$

Answer each question.

1. If an equation does not need the Distributive Property, what should you look for next?
   like terms
2. To solve the equation $5x - 3 = 22$, would you "undo" subtraction or multiplication first?
   subtraction
3. Describe how you would solve $\frac{1}{5}x + 3 = 28$.
   Subtract 3 from both sides, then multiply by 5.

Solve each equation using the procedure shown. Show all your steps.

4. $3n - 1 = 14$ → $n = 5$
5. $3(d - 4) = 9$ → $d = 7$
6. $-4(j + 2) - 3j = 6$ → $j = -2$

RETEACH

When solving multi-step equations, first combine like terms on each side if possible. Then use inverse operations.

	Operations	Solve using Inverse Operations
$4x - 3 = 15$	• x is multiplied by 4. • Then 3 is subtracted.	• Add 3 to both sides. • Then divide both sides by 4.
$\frac{x}{3} + 2 = 9$	• x is divided by 3. • Then 2 is added.	• Add −2 to both sides. • Then multiply both sides by 3.

The order of the inverse operations is the order of operations in reverse.

Solve $2x - 7 + 3x = 13$.

$2x + 3x - 7 = 13$ *Group like terms together.*
$5x - 7 = 13$ *Add like terms.*
$5x - 7 = 13$ *x is multiplied by 5. Then 7 is subtracted.*
$+7 \quad +7$ *Add 7 to both sides.*
$5x = 20$
$\frac{5x}{5} = \frac{20}{5}$ *Divide both sides by 5.*
$x = 4$

Check:
$2x - 7 + 3x = 13$
$2(4) - 7 + 3(4) \stackrel{?}{=} 13$
$8 - 7 + 12 \stackrel{?}{=} 13$
$13 \stackrel{?}{=} 13$ ✓

Solve each equation. Check your answers.

1. $3x - 8 = 4$ → 4
2. $\frac{b}{2} - 4 = 26$ → 60
3. $5y + 4 - 2y = 9$ → $\frac{5}{3}$
4. $14 = 3(x - 2) + 5$ → 5

Write an equation to represent each relationship. Solve each equation.

50. Seven less than twice a number equals 19. $2n - 7 = 19$; $n = 13$

51. Eight decreased by 3 times a number equals 2. $8 - 3n = 2$; $n = 2$

52. The sum of two times a number and 5 is 11. $2n + 5 = 11$; $n = 3$

53. **History** In 1963, Dr. Martin Luther King Jr. began his famous "I have a dream" speech with the words "Five score years ago, a great American, in whose symbolic shadow we stand, signed the Emancipation Proclamation." The proclamation was signed by President Abraham Lincoln in 1863.

a. Using the dates given, write and solve an equation that can be used to find the number of years in a score. $1963 - 5s = 1863$; $s = 20$

b. How many score would represent 60? 3

LINK History

Martin Luther King Jr. entered college at age 15. During his life he earned 3 degrees and was awarded 20 honorary degrees.

Source: lib.lsu.edu

Solve each equation. Check your answer.

54. $3t + 44 = 50$ 2

55. $3(x - 2) = 18$ 8

56. $15 = \frac{c}{3} - 2$ 51

57. $2x + 6.5 = 15.5$ 4.5

58. $3.9w - 17.9 = -2.3$ 4

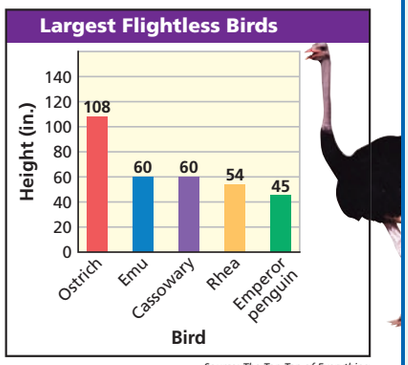
59. $17 = x - 3(x + 1)$

60. $5x + 9 = 39$ 6

61. $15 + 5.5m = 70$ 10

59. −10

Biology **Use the graph for Exercises 62 and 63.**



Source: The Top Ten of Everything

62. The height of an ostrich is 20 inches more than 4 times the height of a kiwi. Write and solve an equation to find the height of a kiwi. Show that your answer is reasonable.

62. $4k + 20 = 108$; 22 in.

63. Five times the height of a kakapo minus 70 equals the height of an emu. Write and solve an equation to find the height of a kakapo. Show that your answer is reasonable.

63. $5k - 70 = 60$; 26 in.

64. The sum of two consecutive whole numbers is 57. What are the two numbers? (*Hint:* Let n represent the first number. Then $n + 1$ is the next consecutive whole number.) 28 and 29

65. Stan's, Mark's, and Wayne's ages are consecutive whole numbers. Stan is the youngest, and Wayne is the oldest. The sum of their ages is 111. Find their ages. Stan: 36; Mark: 37; Wayne: 38

66. The sum of two consecutive even whole numbers is 206. What are the two numbers? (*Hint:* Let n represent the first number. What expression can you use to represent the second number?) 102 and 104

MULTI-STEP TEST PREP

67. **a.** The cost of fighting a certain forest fire is \$225 per acre. Complete the table.

b. Write an equation for the relationship between the cost c of fighting the fire and the number of acres n. $c = 225n$

Cost of Fighting Fire	
Acres	Cost ($)
100	22,500
200	45,000
500	112,500
1000	225,000
1500	337,500
n	$225n$

MULTI-STEP TEST PREP **Exercise 67** involves developing a pattern to write an expression that describes cost. This exercise prepares students for the Multi-Step Test Prep.

Answers

62. Possible answer: the height of an ostrich will be more than 4 times the height of a kiwi. 108 is more than $4(22) = 88$, so 22 in. is reasonable.

63. Possible answer: the height of a kakapo will be more than $\frac{1}{5}$ the height of the emu. 26 is more than $\frac{1}{5}(60) = 12$, so it is a reasonable answer.

(l)© Robert W. Kelley/Time Life Pictures/Getty Images; (b), ©Index Stock/Alamy; (cr),Photodisc/Getty Images

PRACTICE A

PRACTICE C

PRACTICE B

Solve each equation. Check your answers.

1. $-4x + 7 = 11$ $x = -1$
2. $17 = 5y - 3$ $y = 4$
3. $-4 = 2p + 10$ $p = -7$
4. $3m + 4 = 1$ $m = -1$
5. $12.5 = 2g - 3.5$ $g = 8$
6. $-13 = -h - 7$ $h = 6$
7. $-6 = \frac{y}{5} + 4$ $y = -50$
8. $\frac{7}{9} = 2n + \frac{1}{9}$ $n = \frac{1}{3}$
9. $-\frac{4}{5}t + \frac{2}{5} = \frac{2}{3}$ $t = -\frac{1}{3}$
10. $-(x - 10) = 7$ $x = 3$
11. $-2(b + 5) = -6$ $b = -2$
12. $8 = 4(q - 2) + 4$ $q = 3$

13. If $3x - 8 = -2$, find the value of $x - 6$. −4

14. If $-2(3y + 5) = -4$, find the value of $5y$. −5

Answer each of the following.

15. The two angles shown form a right angle. Write and solve an equation to find the value of x.

$3x - 5 + 2x = 90$; 19

16. For her cellular phone service, Vera pays \$32 a month, plus \$0.75 for each minute over the allowed minutes in her plan. Vera received a bill for \$47 last month. For how many minutes did she use her phone beyond the allowed minutes? 20 minutes

PROBLEM SOLVING

Write the correct answer.

1. Stephen belongs to a movie club in which he pays an annual fee of \$39.95 and then rents DVDs for \$0.99 each. In one year, Stephen spent \$55.79. Write and solve an equation to find how many DVDs d he rented. $39.95 + 0.99d = 55.79$; 16 DVDs

2. In 2003, the population of Zimbabwe was about 12.6 million, which was 1 million more than 4 times the population in 1950. Write and solve an equation to find the population p of Zimbabwe in 1950. $12.6 = 4p + 1$; 2.9 million

3. Maggie's brother is three years younger than twice her age. The sum of their ages is 24. How old is Maggie? 9 years old

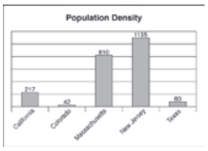
4. Kate is saving to take an SAT prep course that costs \$350. So far, she has saved \$180, and she adds \$17 to her savings each week. How many more weeks must she save to be able to afford the course? 10 weeks

Use the graph below to answer questions 5–7. Select the best answer. The graph shows the population density (number of people per square mile) of various states given in the 2000 census.

5. One seventeenth of Rhode Island's population density minus 17 equals the population density of Colorado. What is Rhode Island's population density?
A 425 C 714
B 697 D 1003

6. One more than sixteen times the population density of New Mexico equals the population density of Texas. To the nearest whole number, what is New Mexico's population density?
F 5 H 13
G 8 J 63

7. Three times the population density of Missouri minus 26 equals the population density of California. What is Missouri's population density?
A 64 C 98
B 81 D 729



CHALLENGE

Many concepts of algebra can be applied to a wide range of geometry problems.

Suppose that you want to design a box. The base of the box will be a square that is 10 inches on each side, and the box will be h inches tall. The surface area of the box (that is, the area of cardboard needed to make the box, assuming no overlap) is given by $4 \cdot 10 \cdot h + 2 \cdot 10 \cdot 10$, or $40h + 200$. For a surface area of 360 square inches, you would solve $40h + 200 = 360$ in order to find the height of the box.

The base of a rectangular box is to be a square that is 10 inches on each side. For each given surface area, find the corresponding height of the box.

1. 360 square inches 4 inches
2. 520 square inches 8 inches
3. 240 square inches 1 inch
4. 560 square inches 9 inches
5. 800 square inches 15 inches
6. 480 square inches 7 inches

Now suppose that you want to design a cylindrical box whose base is a circle with a radius of 5 inches. The surface area of the cylindrical box is given by $50\pi + 10\pi h$.

The base of a cylindrical box will be a circle with a radius of 5 inches. For each given surface area, find the corresponding height of the box.

7. 80π square inches 3 inches
8. 120π square inches 7 inches
9. 110π square inches 6 inches
10. 160π square inches 11 inches
11. 200π square inches 15 inches
12. 90π square inches 4 inches

Another geometric application of two-step equations relates to the interior angles of a polygon. If the polygon has n sides, the sum of the measures of its angles is $180n - 360$ degrees.

For example, in a triangle, $n = 3$, so the measures of the angles add up to 180°. For a trapezoid, $n = 4$, so the measures of the angles add up to 360°.

In the following exercises, the sum of the measures of the interior angles of a polygon is given. Find the number of sides of the polygon.

13. 540° 5 sides
14. 1800° 12 sides
15. 900° 7 sides
16. 2880° 18 sides

TEST PREP DOCTOR

In **Exercise 70,** have students write expressions for each person. Greg: g; Lin: $g + 4$; Fran: $3(g + 4)$ Asking for the number of shirts Greg sold is the same as asking to solve for g. Students who chose **C** may not have distributed 3 through the quantity $(g + 4)$.

Students who chose **F** in **Exercise 71** probably found the correct value of m, but may not have substituted it into $7m - 5$.

Journal

Have students explain to a friend who missed the lesson how to solve a two-step equation. Students should write and solve a word problem as part of their explanation.

ALTERNATIVE ASSESSMENT

Have students create a multi-step equation. Ask students to describe the solution steps and explain why each step is performed. Check that students understand reversing the order of operations applied to the variable.

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1-4 Lesson Quiz

Solve each equation.

1. $4y + 8 = 2$ $-\frac{3}{2}$
2. $\frac{3}{4}a + 14 = 8$ -8
3. $2y + 29 - 8y = 5$ 4
4. $3(x - 9) = 30$ 19
5. $x - (12 - x) = 38$ 25
6. $\frac{z}{6} - \frac{5}{8} = \frac{7}{8}$ 9
7. If $3b - (6 - b) = -22$, find the value of $7b$. -28
8. Josie bought 4 cases of sports drinks for an upcoming meet. After talking to her coach, she bought 3 more cases and spent an additional \$6.95 on other items. Her receipts totaled \$74.15. Write and solve an equation to find how much each case of sports drinks cost.
   $4c + 3c + 6.95 = 74.15$; \$9.60

Also available online

68. **Critical Thinking** The equation $2(m - 8) + 3 = 17$ has more than one solution method. Give at least two different "first steps" to solve this equation.

68. Possible answer: Distribute 2 or subtract 3 from both sides.

69. **Write About It** Write a series of steps that you can use to solve any multi-step equation. **Possible answer: Simplify both sides if necessary. Find the term containing the variable. Undo any addition or subtraction on this term by using inverse operations. Then undo any multiplication or division on this term by using inverse operations.**

TEST PREP

70. Lin sold 4 more shirts than Greg. Fran sold 3 times as many shirts as Lin. In total, the three sold 51 shirts. Which represents the number of shirts Greg sold?

Ⓐ $3g = 51$ Ⓑ $3 + g = 51$ Ⓒ $8 + 5g = 51$ Ⓓ $16 + 5g = 51$

71. If $\frac{4m - 3}{7} = 3$, what is the value of $7m - 5$?

Ⓕ 6 Ⓖ 10.5 Ⓗ 37 Ⓙ 68.5

72. The equation $c = 48 + 0.06m$ represents the cost c of renting a car and driving m miles. Which statement best describes this cost?

Ⓐ The cost is a flat rate of \$0.06 per mile.
Ⓑ The cost is \$0.48 for the first mile and \$0.06 for each additional mile.
Ⓒ The cost is a \$48 fee plus \$0.06 per mile.
Ⓓ The cost is a \$6 fee plus \$0.48 per mile.

73. **Gridded Response** A telemarketer earns \$150 a week plus \$2 for each call that results in a sale. Last week she earned a total of \$204. How many of her calls resulted in sales? **27**

CHALLENGE AND EXTEND

Solve each equation. Check your answer.

74. $\frac{9}{2}x + 18 + 3x = \frac{11}{2}$ $-\frac{5}{3}$

75. $\frac{15}{4}x - 15 = \frac{33}{4}$ $6\frac{1}{5}$

76. $(x + 6) - (2x + 7) - 3x = -9$ **2**

77. $(4x + 2) - (12x + 8) + 2(5x - 3) = 6 + 11$ **14.5**

78. Find a value for b so that the solution of $4x + 3b = -1$ is $x = 2$. **−3**

79. Find a value for b so that the solution of $2x - 3b = 0$ is $x = -9$. **−6**

80. **Business** The formula $p = nc - e$ gives the profit p when a number of items n are each sold at a cost c and expenses e are subtracted.

a. If $p = 2500$, $n = 2000$, and $e = 800$, what is the value of c? **1.65**

b. If $p = 2500$, $n = 1000$, and $e = 800$, what is the value of c? **3.3**

c. **What if...?** If n is divided in half while p and e remain the same, what is the effect on c? ***c* doubles.**

1-5 Algebra LAB

Model Equations with Variables on Both Sides

Algebra tile models can help you understand how to solve equations with variables on both sides.

Use with Solving Equations with Variables on Both Sides

KEY

+ = 1

− = −1

+ = x

− = $-x$

REMEMBER

+ + − = 0

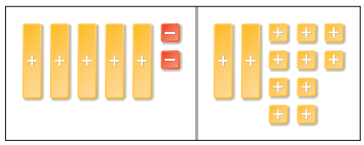
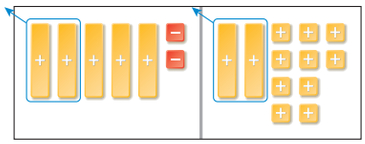
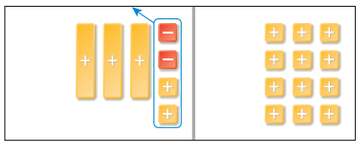
Learn It Online
Lab Resources Online

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. *Also* **CC.9-12.A.REI.3**

Activity

Use algebra tiles to model and solve $5x - 2 = 2x + 10$.

MODEL		ALGEBRA
	*Model $5x - 2$ on the left side of the mat and $2x + 10$ on the right side.* *Remember that $5x - 2$ is the same as $5x + (-2)$.*	$5x - 2 = 2x + 10$
	*Remove 2 x-tiles from both sides. This represents subtracting 2x from both sides of the equation.*	$5x - 2 - 2x = 2x - 2x + 10$ $3x - 2 = 10$
	*Place 2 yellow tiles on both sides. This represents adding 2 to both sides of the equation.* *Remove zero pairs.*	$3x - 2 + 2 = 10 + 2$ $3x = 12$
	Separate each side into 3 equal groups. *Each group is $\frac{1}{3}$ of the side.* *One x-tile is equivalent to 4 yellow tiles.*	$\frac{1}{3}(3x) = \frac{1}{3}(12)$ $x = 4$

Try This

Use algebra tiles to model and solve each equation.

1. $3x + 2 = 2x + 5$ **3**
2. $5x + 12 = 2x + 3$ **−3**
3. $9x - 5 = 6x + 13$ **6**
4. $x = -2x + 9$ **3**

Teacher to Teacher

Sometimes students wonder how to subtract tiles from both sides if there are not enough tiles to take away.

"How can I take away 12 when I have only 3?" a student might ask about **Problem 2** in *Try This*.

Show students how to subtract from both sides by adding negative tiles and then removing zero pairs. I find that using this method really helps students remember that subtraction is defined as adding an opposite.

Sue Pippen
Darien, IL

Algebra LAB **Organizer**

Use with the lesson Solving Equations with Variables on Both Sides

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use algebra tiles to model and solve equations with variables on both sides.

Materials: algebra tiles

INTERACTIVE **Online Edition**
Algebra Tiles

Countdown

Resources

LAB ***Algebra Lab Activities***
Lab Recording Sheet

Teach

Discuss

Explain that the first objective is to get x-tiles on only one side. Then solve using methods previously learned.

Close

Key Concept

You may add or remove the same number of x-tiles on both sides.

Assessment

Journal Have students explain each step in solving one of the equations in *Try This*.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. *Also* **CC.9-12.A.REI.3**

Learn It Online
State Resources Online

1-5 Organizer

Pacing: Traditional $1\frac{1}{2}$ days
Block $\frac{3}{4}$ day

Objective: Solve equations in one variable that contain variable terms on both sides.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Simplify.

1. $4x - 10x$ $-6x$
2. $-7(x - 3)$ $-7x + 21$
3. $\frac{1}{3}(6x + 9)$ $2x + 3$
4. $15 - (x - 2)$ $17 - x$

Solve.

5. $3x + 2 = 8$ 2
6. $\frac{1}{3}x - \frac{1}{3} = 9$ 28

Also available online

Math Humor

Q: Why did the variable add its opposite?

A: To get to the other side.

COMMON CORE

CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution….

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *Also* **CC.9-12.A.CED.1***

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COMMON CORE

1-5

Solving Equations with Variables on Both Sides

CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution…. *Also* **CC.9-12.A.REI.3, CC.9-12.A.CED.1***

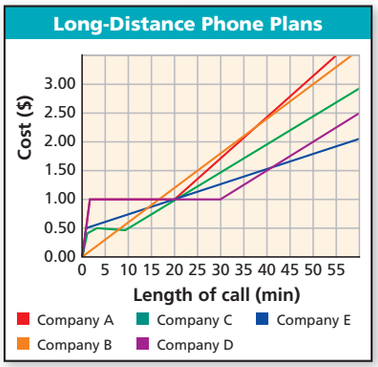
Objective
Solve equations in one variable that contain variable terms on both sides.

Vocabulary
identity

Why learn this?
You can compare prices and find the best value.

Many phone companies offer low rates for long-distance calls without requiring customers to sign up for their services. To compare rates, solve an equation with variables on both sides.

To solve an equation like this, use inverse operations to "collect" variable terms on one side of the equation.



EXAMPLE 1 Solving Equations with Variables on Both Sides

Solve each equation.

A $7k = 4k + 15$

$7k = 4k + 15$
$-4k \quad -4k$
$3k = 15$
$\frac{3k}{3} = \frac{15}{3}$
$k = 5$

To collect the variable terms on one side, subtract 4k from both sides.

Since k is multiplied by 3, divide both sides by 3 to undo the multiplication.

B $5x - 2 = 3x + 4$

$5x - 2 = 3x + 4$
$-3x \quad -3x$
$2x - 2 = 4$
$+2 \quad +2$
$2x = 6$
$\frac{2x}{2} = \frac{6}{2}$
$x = 3$

To collect the variable terms on one side, subtract 3x from both sides.

Since 2 is subtracted from 2x, add 2 to both sides to undo the subtraction.

Since x is multiplied by 2, divide both sides by 2 to undo the multiplication.

Check

$5x - 2 = 3x + 4$	
$5(3) - 2$	$3(3) + 4$
$15 - 2$	$9 + 4$
13	13 ✓

To check your solution, substitute 3 for x in the original equation.

Helpful Hint
Equations are often easier to solve when the variable has a positive coefficient. Keep this in mind when deciding on which side to "collect" variable terms.

CHECK IT OUT! **Solve each equation. Check your answer.**

1a. $4b + 2 = 3b$ -2

1b. $0.5 + 0.3y = 0.7y - 0.3$ 2

To solve more complicated equations, you may need to first simplify by using the Distributive Property or combining like terms.

1 Introduce

EXPLORATION

Solving Equations with Variables on Both Sides

You will need a graphing calculator for this Exploration.

A receptionist is buying plants to decorate the lobby of an office building. The receptionist wants to have the plants delivered. Flower City charges $50 plus $6 per plant. Plants for Less charges $8 per plant.

1. Write an expression that gives the cost of buying plants from Flower City. $50 + 6x$
2. Write an expression that gives the cost of buying plants from Plants for Less. $8x$
3. On your graphing calculator, press Y=. Then enter the expressions from Problems 1 and 2 in Y_1 and Y_2.
4. Use your calculator's table feature to find out what number of plants costs the same from both companies. 25

THINK AND DISCUSS

5. **Explain** which company the receptionist should use if 20 plants are needed for the lobby. Plants for Less
6. **Describe** another method you could use to find the number of plants that costs the same from both companies. Possible answer: Set the expressions equal to each other to get the equation $50 + 6x = 8x$ and then solve for x.

Motivate

Draw a balance scale on the board. On one side, draw a 6-ounce weight and an unknown weight. On the other side, draw a 4-ounce weight and two unknown weights. Ask students what the unknown weight must be in order to balance the scale. 2 oz

Remind students that adding or removing an equal amount on both sides of a balanced scale will maintain the balance. This is true even when the weight being added or removed is unknown.

Explorations and answers are provided in your online resources.

EXAMPLE 2 Simplifying Each Side Before Solving Equations

Solve each equation.

A $2(y+6)=3y$

$2(y+6)=3y$ — *Distribute 2 to the expression in parentheses.*

$2(y)+2(6)=3y$

$2y+12=3y$

$-2y \quad -2y$ — *To collect the variable terms on one side, subtract 2y from both sides.*

$12=y$

Check $2(y+6)=3y$

$2(12+6)$	$3(12)$
$2(18)$	36
36	36 ✓

To check your solution, substitute 12 for y in the original equation.

B $2k-5=3(1-2k)$

$2k-5=3(1-2k)$ — *Distribute 3 to the expression in parentheses.*

$2k-5=3(1)-3(2k)$

$2k-5=3-6k$

$+6k \quad +6k$ — *To collect the variable terms on one side, add 6k to both sides.*

$8k-5=3$

$+5 \quad +5$ — *Since 5 is subtracted from 8k, add 5 to both sides.*

$8k=8$

$\frac{8k}{8}=\frac{8}{8}$ — *Since k is multiplied by 8, divide both sides by 8.*

$k=1$

C $3-5b+2b=-2-2(1-b)$

$3-5b+2b=-2-2(1-b)$

$3-5b+2b=-2-2(1)-2(-b)$ — *Distribute −2 to the expression in parentheses.*

$3-5b+2b=-2-2+2b$ — *Combine like terms.*

$3-3b=-4+2b$ — *Add 3b to both sides.*

$+3b \quad +3b$

$3=-4+5b$ — *Since −4 is added to 5b, add 4 to both sides.*

$+4 \quad +4$

$7=5b$ — *Since b is multiplied by 5, divide both sides by 5.*

$\frac{7}{5}=\frac{5b}{5}$

$1.4=b$

CHECK IT OUT! **Solve each equation. Check your answer.**

2a. $\frac{1}{2}(b+6)=\frac{3}{2}b-1$ 4

2b. $3x+15-9=2(x+2)$ −2

An **identity** is an equation that is always true, no matter what value is substituted for the variable. The solutions of an identity are all real numbers. Some equations are always false. These equations have no solutions.

COMMON ERROR ALERT

Students may try to combine the like terms that they see in an equation. For example, $5x-2=3x+4$ becomes $8x-2=4$. Remind students that they must instead "undo" the addition of $3x$ on the right side.

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Additional Examples

Example 1

Solve $7n-2=5n+6$. 4

Example 2

Solve $4-6a+4a=-1-5(7-2a)$. $\frac{10}{3}$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- How do you decide which variable term to add to or subtract from one side of the equation?

EXAMPLE 2

- How are the equations in the second example different from those in the first example?
- How do you know when one side of an equation is not simplified?
- How do you simplify each side of an equation?
- Why is it important to simplify each side of an equation before you add or subtract terms on each side?

Teaching Tip **Visual** To remind students to include negative values in their multiplication when distributing, as in **Example 2C,** circle the negative number that is to be distributed, perhaps with a colored pencil (MK).

2 Teach

Guided Instruction

Before solving equations with variables on both sides, review the steps used to solve equations with a variable on one side. Then review how to combine like terms and how to simplify using the Distributive Property. Show students that adding or subtracting the variable terms on each side, before adding and subtracting constant terms, results in a one-step equation like those from earlier lessons.

Reaching All Learners

Through Cooperative Learning

Give each student a card with an expression such as $2x+4$ or $3x-7$ written on it. Have students pair up and solve the equation that is formed by setting their expressions equal to each other. Students should pair up with as many students as time allows. For advanced learners, expressions should require simplifying.

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Additional Examples

Example 3

Solve each equation.

A. $10 - 5x + 1 = 7x + 11 - 12x$
all real numbers

B. $12x - 3 + x = 5x - 4 + 8x$
no solution

Example 4

Jon and Sara are planting tulip bulbs. Jon has planted 60 bulbs and is planting at a rate of 44 bulbs per hour. Sara has planted 96 bulbs and is planting at a rate of 32 bulbs per hour. In how many hours will Jon and Sara have planted the same number of bulbs? How many bulbs will that be? 3 h; 192 bulbs

Also available online

Teaching Tip **Number Sense** Encourage students to add and subtract variable terms so that the result is a positive coefficient. Often this means to subtract the variable term with the lesser coefficient on both sides.

Mathematical Practices **Questioning Strategies**

EXAMPLE 3

- What is the solution of an equation if the last line is a true statement? a false statement?

EXAMPLE 4

- Which phrase in the question indicates what the variable will represent?

EXAMPLE 3 Infinitely Many Solutions or No Solutions

Solve each equation.

A $x + 4 - 6x = 6 - 5x - 2$

$x + 4 - 6x = 6 - 5x - 2$ *Identify like terms.*

$4 - 5x = 4 - 5x$ *Combine like terms on the left and the right.*

$+5x \quad +5x$ *Add 5x to both sides.*

$4 = 4$ ✓ *True statement*

The equation $x + 4 - 6x = 6 - 5x - 2$ is an identity. All values of x will make the equation true. All real numbers are solutions.

Writing Math

The solution set for Example 3B is an empty set—it contains no elements. The empty set can be written as ∅ or {}.

B $-8x + 6 + 9x = -17 + x$

$-8x + 6 + 9x = -17 + x$ *Identify like terms.*

$x + 6 = -17 + x$ *Combine like terms.*

$-x \quad -x$ *Subtract x from both sides.*

$6 = -17$ ✗ *False statement*

The equation $-8x + 6 + 9x = -17 + x$ is always false. There is no value of x that will make the equation true. There are no solutions.

Check It Out! **Solve each equation.**

3a. $4y + 7 - y = 10 + 3y$
no solution

3b. $2c + 7 + c = -14 + 3c + 21$
all real numbers

EXAMPLE 4 *Consumer Application*

The long-distance rates of two phone companies are shown in the table. How long is a call that costs the same amount no matter which company is used? What is the cost of that call?

Phone Company	Charges
Company A	36¢ plus 3¢ per minute
Company B	6¢ per minute

Let m represent minutes, and write expressions for each company's cost.

When is | 36¢ | plus | 3¢ per minute | times number of minutes | the same as | 6¢ per minute | times number of minutes | ?

$36 + 3 (m) = 6 (m)$

$36 + 3m = 6m$

$-3m \quad -3m$

$36 = 3m$

$\frac{36}{3} = \frac{3m}{3}$

$12 = m$

To collect the variable terms on one side, subtract 3m from both sides.

Since m is multiplied by 3, divide both sides by 3 to undo the multiplication.

The charges will be the same for a 12-minute call using either phone service. To find the cost of this call, evaluate either expression for $m = 12$:

$36 + 3m = 36 + 3(12) = 36 + 36 = 72 \qquad 6m = 6(12) = 72$

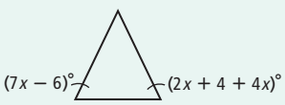
The cost of a 12-minute call through either company is 72¢.

Check It Out! **4.** Four times Greg's age, decreased by 3 is equal to 3 times Greg's age, increased by 7. How old is Greg? 10 years old

3 Close

Summarize

Draw the triangle below on the board:



Tell students that the triangle is isosceles and that the labeled angles have the same measure. Have them write and solve an equation for x, explaining each step. Then have them find the measure of each angle.

$x = 10$; each angle measures 64°.

Ongoing Assessment and Intervention

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
2-4 Lesson Quiz, TE
Alternative Assessment, TE

THINK AND DISCUSS

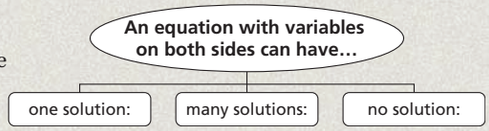
MATHEMATICAL PRACTICES

1. Tell which of the following is an identity. Explain your answer.

a. $4(a+3)-6=3(a+3)-6$ **b.** $8.3x-9+0.7x=2+9x-11$

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of an equation that has the indicated number of solutions.



1-5 Exercises

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GUIDED PRACTICE

1. Vocabulary How can you recognize an identity?

Solve each equation. Check your answer.

SEE EXAMPLE 1

2. $2c-5=c+4$ 9

3. $8r+4=10+2r$ 1

4. $2x-1=x+11$ 12

5. $28-0.3y=0.7y-12$ 40

SEE EXAMPLE 2

6. $-2(x+3)=4x-3$ $-\frac{1}{2}$

7. $3c-4c+1=5c+2+3$ $-\frac{2}{3}$

8. $5+3(q-4)=2(q+1)$ 9

9. $5-(t+3)=-1+2(t-3)$ 3

SEE EXAMPLE 3

10. $7x-4=-2x+1+9x-5$

11. $8x+6-9x=2-x-15$ no solution

12. $6y=8-9+6y$ no solution

13. $6-2x-1=4x+8-6x-3$ all real numbers

SEE EXAMPLE 4

14. Consumer Economics A house-painting company charges $376 plus $12 per hour. Another painting company charges $280 plus $15 per hour.

a. How long is a job for which both companies will charge the same amount? 32 h

b. What will that cost be? $760

PRACTICE AND PROBLEM SOLVING

Solve each equation. Check your answer.

15. $7a-17=4a+1$ 6

16. $2b-5=8b+1$ −1

17. $4x-2=3x+4$ 6

18. $2x-5=4x-1$ −2

19. $8x-2=3x+12.25$ 2.85

20. $5x+2=3x$ −1

21. $3c-5=2c+5$ 10

22. $-17-2x=6-x$ −23

23. $3(t-1)=9+t$ 6

24. $5-x-2=3+4x+5$ −1

25. $2(x+4)=3(x-2)$ 14

26. $3m-10=2(4m-5)$ 0

27. $5-(n-4)=3(n+2)$ $\frac{3}{4}$

28. $6(x+7)-20=6x$

29. $8(x+1)=4x-8$ −4

30. $x-4-3x=-2x-3-1$

31. $-2(x+2)=-2x+1$

32. $2(x+4)-5=2x+3$

28. no solution
30. all real numbers
31. no solution
32. all real numbers

Answers

1. Possible answer: After simplifying, the expressions on either side of the equal sign are the same.
10. all real numbers

Answers to *Think and Discuss*

1. Equation b, $8.3x-9+0.7x=2+9x-11$, is an identity. The equation simplifies to $9x-9=9x-9$, which is true for all values of x.
2. See Additional Answers.

1-5 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–2

Basic 15–30, 35–37, 38–52 even

Average 15–30, 38–52 even, 54, 60, 63

Advanced 15–30, 42–52 even, 54, 64, 65, 68, 69

If you finished Examples 1–4

Basic 15–52, 54, 55, 57, 59–63

Average 15–37, 38–52 even, 53–63, 66–69

Advanced 15–34, 36, 42–52 even, 54–71

Homework Quick Check

Quickly check key concepts.

Exercises: 20, 26, 32, 34, 36, 54

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 14, 33, 52, 54, 56

Reason abstractly and quantitatively. Exercises 55, 69–71

Construct viable arguments and critique the reasoning of others. Exercise 57

Model with mathematics. Exercises 53, 58

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Teaching Tip **Geometry** For **Exercise 53,** remind students that perimeter is the distance around a figure. Students may benefit from writing $P_1 = P_2$, and then substituting P_1 and P_2 with expressions for each perimeter.

MULTI-STEP TEST PREP **Exercise 54** involves writing an equation to represent a real-world relationship. This exercise prepares students for the Multi-Step Test Prep.

Answers

52a. Possible answer: For 100 mi, Rapid Rental charges \$40 + \$15 + \$25 = \$80. Capital Cars charges \$45 + \$35 = \$80.

54a. Possible answer: the total acreage in the next 2 days will be about 100 more than 420, so 540 is reasonable.

Independent Practice

For Exercises	See Example
15–22	1
23–29	2
30–32	3
33	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

34. $3(x + 4) = 18 + x$; $x = 3$

35. $x - 30 = 14 - 3x$; $x = 11$

36. $2x - 2 = x + 64$; $x = 66$

51. no solution

33. Sports Justin and Tyson are beginning an exercise program to train for football season. Justin weighs 150 lb and hopes to gain 2 lb per week. Tyson weighs 195 lb and hopes to lose 1 lb per week.

a. If the plan works, in how many weeks will the boys weigh the same amount? 15

b. What will that weight be? 180 lb

Write an equation to represent each relationship. Then solve the equation.

34. Three times the sum of a number and 4 is the same as 18 more than the number.

35. A number decreased by 30 is the same as 14 minus 3 times the number.

36. Two less than 2 times a number is the same as the number plus 64.

Solve each equation. Check your answer.

37. $2x - 2 = 4x + 6$ **–4**

38. $3x + 5 = 2x + 2$ **–3**

39. $4x + 3 = 5x - 4$ **7**

40. $-\frac{2}{5}p + 2 = \frac{1}{5}p + 11$ **–15**

41. $5x + 24 = 2x + 15$ **–3**

42. $5x - 10 = 14 - 3x$ **3**

43. $12 - 6x = 10 - 5x$ **2**

44. $5x - 7 = -6x - 29$ **–2**

45. $1.8x + 2.8 = 2.5x + 2.1$ **1**

46. $2.6x + 18 = 2.4x + 22$ **20**

47. $1 - 3x = 2x + 8$ $-\frac{7}{5}$

48. $\frac{1}{2}(8 - 6h) = h$ **1**

49. $3(x + 1) = 2x + 7$ **4**

50. $9x - 8 + 4x = 7x + 16$ **4**

51. $3(2x - 1) + 5 = 6(x + 1)$

52. Travel Rapid Rental Car company charges a \$40 rental fee, \$15 for gas, and \$0.25 per mile driven. For the same car, Capital Cars charges \$45 for rental and gas and \$0.35 per mile.

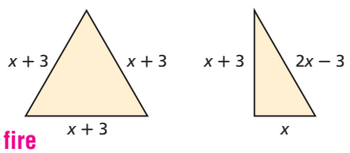
a. Find the number of miles for which the companies' charges will be the same. Then find that charge. Show that your answers are reasonable. 100 mi; \$80

b. The Barre family estimates that they will drive about 95 miles during their vacation to Hershey, Pennsylvania. Which company should they rent their car from? Explain. Capital Cars; the cost would be less.

c. **What if...?** The Barres have extended their vacation and now estimate that they will drive about 120 miles. Should they still rent from the same company as in part **b**? Why or why not? No; now Rapid Rental would be cheaper.

d. Give a general rule for deciding which company to rent from.
Less than 100 mi—use Capital; more than 100 mi—use Rapid.

53. Geometry The triangles shown have the same perimeter. What is the value of x? 9



54c. $80d$

d. $420 + 60d = 80d$; $d = 21$ days; the number of days it will take the firefighters to put out the fire

MULTI-STEP TEST PREP

54. a. A fire currently covers 420 acres and continues to spread at a rate of 60 acres per day. How many total acres will be covered in the next 2 days? Show that your answer is reasonable. 540 acres

b. Write an expression for the total area covered by the fire in d days. $420 + 60d$

c. The firefighters estimate that they can put out the fire at a rate of 80 acres per day. Write an expression for the total area that the firefighters can put out in d days.

d. Set the expressions in parts **b** and **c** equal. Solve for d. What does d represent?

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PRACTICE A
PRACTICE C
PRACTICE B

Solve each equation. Check your answers.

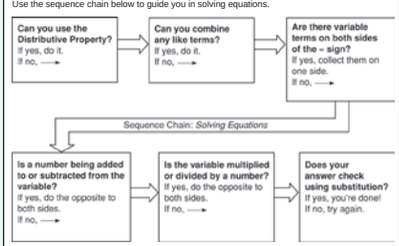
1. $3d + 8 = 2d - 17$ — $d = -25$
2. $2n - 7 = 5n - 10$ — $n = 1$
3. $p - 15 = 13 - 6p$ — $p = 4$
4. $-t + 5 = t - 19$ — $t = 12$
5. $15x - 10 = -9x + 2$ — $x = \frac{1}{2}$
6. $1.8r + 9 = -5.7r - 6$ — $r = -2$
7. $2y + 3 = 3(y + 7)$ — $y = -18$
8. $4n + 6 - 2n = 2(n + 3)$ — all real numbers
9. $6m - 8 = 2 + 9m - 1$ — $m = -3$
10. $-v + 5 + 6v = 1 + 5v + 3$ — no solution
11. $2(3b - 4) = 8b - 11$ — $b = \frac{3}{2}$
12. $5(r - 1) = 2(r - 4) - 6$ — $r = -3$

Answer each of the following.

13. Janine has job offers at two companies. One company offers a starting salary of \$28,000 with a raise of \$3000 each year. The other company offers a starting salary of \$36,000 with a raise of \$2000 each year.
    a. After how many years would Janine's salary be the same with both companies? 8 years
    b. What would that salary be? \$52,000
14. Xian and his cousin both collect stamps. Xian has 56 stamps, and his cousin has 80 stamps. Both have recently joined different stamp-collecting clubs. Xian's club will send him 12 new stamps per month, and his cousin's club will send him 8 new stamps per month.
    a. After how many months will Xian and his cousin have the same number of stamps? 6 months
    b. How many stamps will that be? 128 stamps

READING STRATEGIES

Use the sequence chain below to guide you in solving equations.



Answer each question.

1. What is the first thing to look for when solving an equation?
   Look for an opportunity to use the Distributive Property.
2. What should you do before collecting the variables on one side of the equal sign?
   Combine any like terms.
3. Describe the first step in solving the equation $3x - 4 = 2x + 19$.
   Collect the variable terms on one side of the equal sign.

Solve each equation using the sequence chain.

4. $7p - 2 = 9p + 10$ — $p = -6$
5. $8 + 4x = 3(x - 1) + 12$ — $x = 1$
6. $-2(t + 2) + 5t = 6t + 2$ — $t = -2$

RETEACH

Variables must be collected on the same side of the equation before the equation can be solved.

Solve $10x = 2x - 16$.

		Check:
$10x = 2x - 16$		$10x = 2x - 16$
$\frac{-2x}{8x} \quad \frac{-2x}{-16}$	Add $-2x$ to both sides.	$10(-2) \stackrel{?}{=} 2(-2) - 16$
		$-20 \stackrel{?}{=} -4 - 16$
$\frac{8x}{8} = \frac{-16}{8}$	Divide both sides by 8.	$-20 \stackrel{?}{=} -20$ ✓
$x = -2$		

Solve $3x = 5(x + 2)$.

		Check:
$3x = 5x + 10$	Distribute.	$3x = 5(x + 2)$
$\frac{-5x}{-2x} \quad \frac{-5x}{10}$	Add $-5x$ to both sides.	$3(-5) \stackrel{?}{=} 5(-5 + 2)$
		$-15 \stackrel{?}{=} 5(-3)$
$\frac{-2x}{-2} = \frac{10}{-2}$	Divide both sides by -2.	$-15 \stackrel{?}{=} -15$ ✓
$x = -5$		

Write the first step you would take to solve each equation.

1. $3x + 2 = 7x$ Possible answers: add $-3x$ to each side, add $-7x$ to each side
2. $-4x - 6 = -10x$ Possible answers: add $4x$ to each side, add $10x$ to each side
3. $15x + 7 = -3x$ Possible answers: add $3x$ to each side, add $-15x$ to each side

Solve each equation. Check your answer.

4. $4x + 2 = 5(x + 10)$ — -48
5. $-10 + y + 3 = 4y - 13$ — 2
6. $3(t + 7) + 2 = 6t - 2 + 2t$ — 5

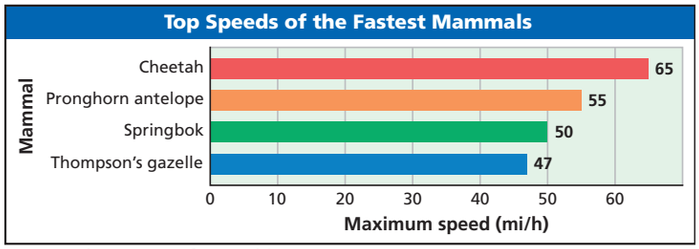
55. Critical Thinking Write an equation with variables on both sides that has no solution. **Possible answer: $2x + 6 = x + 5 + x$**

56. Biology The graph shows the maximum recorded speeds of the four fastest mammals.

Biology

A cheetah's body is well designed for fast running. Its tail acts like a boat's rudder to help it make sharp turns. Its spine acts like a spring to propel it forward.

Source: www.cheetahspot.com



Source: The Top 10 of Everything

a. Write an expression for the distance in miles that a Thompson's gazelle can run at top speed in x hours. **$47x$**

b. Write an expression for the distance in miles that a cheetah can run at top speed in x hours. **$65x$**

56d. $\frac{1}{18}$ h, or about $3\frac{1}{3}$ min

56e. No; the cheetah can only maintain its top speed for about 0.003 h, or about 11 s.

c. A cheetah and a Thompson's gazelle are running at their top speeds. The cheetah is one mile behind the gazelle. Write an expression for the distance the cheetah must run to catch up with the gazelle. **$47x + 1$**

d. Write and solve an equation that represents how long the cheetah will have to run at top speed to catch up with the gazelle. **$47x + 1 = 65x$**

e. A cheetah can maintain its top speed for only 300 yards. Will the cheetah be able to catch the gazelle? Explain.

57. Write About It Write a series of steps that you can use to solve any equation with variables on both sides.

Possible answer: Simplify both sides if necessary. Collect the variable terms on one side by using inverse operations. Then isolate the variable using inverse operations.

TEST PREP

58. Lindsey's monthly magazine subscription costs \$1.25 per issue. Kenzie's monthly subscription costs \$1.50 per issue, but she received her first 2 issues free. Which equation can be used to find the number of months after which the girls will have paid the same amount?

Ⓐ $1.25m = 1.50m - 2$ Ⓒ $1.25m = 1.50(m - 2)$

Ⓑ $1.25m = 1.50m - 2m$ Ⓓ $1.25m = 3m - 1.50$

59. What is the numerical solution of the equation *7 times a number equals 3 less than 5 times that number*?

Ⓕ -1.5 Ⓖ 0.25 Ⓗ $\frac{2}{3}$ Ⓙ 4

60. Three packs of markers cost \$9.00 less than 5 packs of markers. Which equation best represents this situation?

Ⓐ $5x + 9 = 3x$ Ⓑ $3x + 9 = 5x$ Ⓒ $3x - 9 = 5x$ Ⓓ $9 - 3x = 5x$

61. Nicole has \$120. If she saves \$20 per week, in how many days will she have \$500?

Ⓕ 19 Ⓖ 25 Ⓗ 133 Ⓙ 175

62. Gridded Response Solve $-2(x - 1) + 5x = 2(2x - 1)$. **4**

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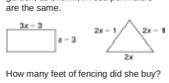
PROBLEM SOLVING

Write the correct answer.

1. Claire purchased just enough fencing to border either a rectangular or triangular garden, as shown, whose perimeters are the same.



How many feet of fencing did she buy?
28 feet

2. Celia and Ryan are starting a nutrition program. Celia currently consumes 1200 calories a day and will increase that number by 100 calories each day. Ryan currently consumes 3230 calories a day and will decrease that number by 190 each day. They will continue this pattern until they are both consuming the same number of calories per day. In how many days will that be?
7 days

3. A moving company charges \$800 plus \$16 per hour. Another moving company charges \$720 plus \$21 per hour. How long is a job that costs the same no matter which company is used?
16 hours

4. Aaron needs to take out a loan to purchase a motorcycle. At one bank, he would pay \$2500 initially and \$150 each month for the loan. At another bank, he would pay \$3000 initially and \$125 each month. After how many months will the two loan payments be the same?
20 months

Use the table below to answer questions 5–7. Select the best answer.
The table shows the membership fees of three different gyms.

Gym	Fees
Workout Now	\$200 plus \$45 per month
Community Gym	\$50 plus \$55 per month
Ultra Sports Club	\$20 plus \$60 per month

5. After how many months will the fees for Workout Now and Community Gym be the same?
A 2.5 C 25
B 15 D 30

6. Sal joined Workout Now for the number of months found in problem 5. How much did he pay?
F \$695 H \$1325
G \$875 J \$1550

7. After how many months will the fees for Workout Now and Ultra Sports Club be the same?
A 7 **C** 12
B 10 D 15

CHALLENGE

A linear equation in one variable can take on many different appearances. Each of the equations below is an equation in one variable.

$2x - 5 = 12$ $2(x + 1) = 5$ $3x - 4 = -2x + 1$

Although these equations look different, you can transform each of them into the standard form for a linear equation in one variable.

In Exercises 1–4, each equation has the form $ax + b = 0$. Solve each equation.

1. $3x + 7 = 0$ **$x = -\frac{7}{3}$**
2. $-2x - 8 = 0$ **$x = -4$**
3. $\frac{2}{3}x - 5 = 0$ **$x = 7\frac{1}{2}$**
4. $-4.1x - 8.2 = 0$ **$x = -2$**

You can always transform a given linear equation in one variable into the form $ax + b = 0$.

In Exercises 5–8, write each equation in the form $ax + b = 0$. Do not solve the resulting equation.

5. $3(x + 5) = x + 21$
$2x + (-6) = 0$ or $-2x + 6 = 0$
6. $-2(-4 - 6x) = -10 + 3x$
$9x + 18 = 0$ or $-9x + (-18) = 0$
7. $-7(-3 - x) = -2(3 - x)$
$5x + 27 = 0$ or $-5x + (-27) = 0$
8. $8 - 2(x - 5) = x - 3$
$3x + (-21) = 0$ or $-3x + 21 = 0$

9. a. Solve $ax + b = 0$ for x. **$x = -\frac{b}{a}$**

b. Explain how to write your answer to part **a** as a formula that you can use to solve a linear equation in x.
After writing the equation in the form $ax + b = 0$, substitute the values of a and b into the formula found in part a.

Solve the specified equation by using your formula from Exercise 9.

10. Exercise 5 **$x = 3$**
11. Exercise 6 **$x = -2$**
12. Exercise 7 **$x = -5\frac{2}{5}$**
13. Exercise 8 **$x = 7$**

14. a. Let $ax + b = 0$, where $a = 0$ and $b = 0$. How many solutions does the equation have? **infinitely many solutions**

b. Let $ax + b = 0$, where $a = 0$ and $b \neq 0$. How many solutions does the equation have? **no solution**

COMMON ERROR ALERT

In **Exercise 51,** students may write 0 instead of *no solution*, thinking that they are writing "zero solutions." Remind students that no solution means that there is no number, including zero, that will make the statement true.

TEST PREP DOCTOR

In **Exercise 58,** only Kenzie's cost (the right side) needs to be analyzed. Have students simplify the right side of **C** so that every choice is in simplified form. Tell students that the cost of two issues at \$1.50 each is \$3.00. This amount is subtracted from the total cost of m issues at \$1.50 per issue.

If students chose **C** in **Exercise 60,** they are probably subtracting \$9 from the lesser cost. Tell students to replace the word *cost* with *is the same as*, to see where the equal sign should go. The correct choice, **B,** is the result of then adding 9 to both sides.

CHALLENGE AND EXTEND

Solve each equation.

63. $4x + 2[4 - 2(x + 2)] = 2x - 4$ **2**

64. $\frac{x+5}{2} + \frac{x-1}{2} = \frac{x-1}{3} - \frac{7}{2}$

65. $\frac{2}{3}w - \frac{1}{4} = \frac{2}{3}\left(w - \frac{1}{4}\right)$ **no solution**

66. $-5 - 7 - 3f = -f - 2(f + 6)$ **all real numbers**

67. $\frac{2}{3}x + \frac{1}{2} = \frac{3}{5}x - \frac{5}{6}$ **−20**

68. $x - \frac{1}{4} = \frac{x}{3} + 7\frac{3}{4}$ **12**

69. Find three consecutive integers such that twice the greatest integer is 2 less than 3 times the least integer. **6, 7, 8**

70. Find three consecutive integers such that twice the least integer is 12 more than the greatest integer. **14, 15, 16**

71. Rob had twice as much money as Sam. Then Sam gave Rob 1 quarter, 2 nickels, and 3 pennies. Rob then gave Sam 8 dimes. If they now have the same amount of money, how much money did Rob originally have? Check your answer. **$1.68**

Career Path

Learn It Online
Career Resources Online

Beth Simmons
Biology major

Q: **What math classes did you take in high school?**
A: Algebra 1 and 2, Geometry, and Precalculus

Q: **What math classes have you taken in college?**
A: Two calculus classes and a calculus-based physics class

Q: **How do you use math?**
A: I use math a lot in physics. Sometimes I would think a calculus topic was totally useless, and then we would use it in physics class! In biology, I use math to understand populations.

Q: **What career options are you considering?**
A: When I graduate, I could teach, or I could go to graduate school and do more research. I have a lot of options.

© Rob Melnychuk/gettyimages

Journal

Have students explain the difference between solving an equation that has all real numbers as solutions and solving an equation that has no solutions.

ALTERNATIVE ASSESSMENT

Have students write and solve

- an equation with variables on both sides,
- an equation with variables on both sides that needs to be simplified on both sides before solving,
- an equation with infinitely many solutions, and
- an equation with no solutions.

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1-5 Lesson Quiz

Solve each equation.

1. $7x + 2 = 5x + 8$ 3

2. $4(2x - 5) = 5x + 4$ 8

3. $6 - 7(a + 1) = -3(2 - a)$ $\frac{1}{2}$

4. $4(3x + 1) - 7x = 6 + 5x - 2$ all real numbers

5. $\frac{2}{3}(3x + 9) = 8x$ 1

6. A painting company charges $250 base plus $16 per hour. Another painting company charges $210 base plus $18 per hour. How long is a job for which the two companies costs are the same? 20 h

Also available online

1-5 Technology LAB

Solve Equations Graphically

You can use graphs to solve equations. As you complete this activity, you will explore connections between graphs and equations.

Use with Lesson Solving Equations with Variables on Both Sides

Activity 1

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

CC.9-12.A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, ….*

Use a graphing calculator to solve $x + 1 = 3x - 5$.

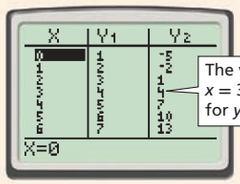
1. Write the equation $x + 1 = 3x - 5$ as two separate equations:

$$y = x + 1$$
$$y = 3x - 5$$

2. Press Y=. Enter the first equation, $y = x + 1$, in Y_1 and the second equation, $y = 3x - 5$, in Y_2.

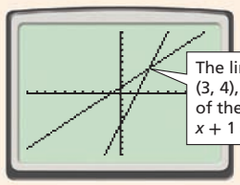
3. Press 2nd TABLE (GRAPH) to use the **TABLE** function.

4. Scroll through the values using ▲ and ▼. Look for values where Y_1 and Y_2 are equal, and then find the corresponding *x*-value. This *x*-value is the solution of the equation.



The value of y at $x = 3$ is the same for y_1 and y_2.

5. Press GRAPH and verify where the lines intersect.



The lines intersect at (3, 4), so the solution of the equation $x + 1 = 3x - 5$ is $x = 3$.

6. You can check your answer by solving the equation algebraically.

$$\begin{aligned} x + 1 &= 3x - 5 \\ x - x + 1 &= 3x - x - 5 \\ 1 &= 2x - 5 \\ 1 + 5 &= 2x - 5 + 5 \\ 6 &= 2x \\ \frac{6}{2} &= \frac{2x}{2} \\ 3 &= x \checkmark \end{aligned}$$

Technology LAB Organizer

Use with the *Solving Equations with Variables on Both Sides*

Pacing:
Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Solve equations with variables on both sides by graphing.

Materials: graphing calculator

Teach

Discuss

Each side of the equation is entered into the **Y=** editor. No matter which side of the equation is entered into **Y1** and which side is entered into **Y2**, the intersection of the lines will remain the same.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.*

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Close

Key Concept

Equations with variables on both sides can be solved by graphing each side of the equation and finding the x-value of the graphs' intersection point.

Assessment

Journal Have students explain the meaning of the intersection of two lines and the difference between the coordinates of the intersection and the solution of the original equation.

Try This

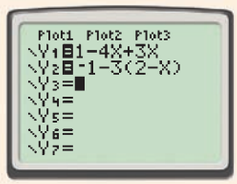
Solve each equation by graphing.

1. $2x + 1 = x + 8$ **7**
2. $4x - 3 = 2x + 9$ **6**
3. $2x - 5 = x - 9$ **–4**
4. $-x + 3 = 5 - 2x$ **2**
5. $x - 10 = -3x + 2$ **3**
6. $-6x + 5 = -x$ **1**

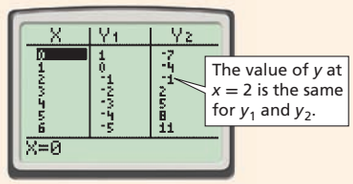
Activity 2

Use a graphing calculator to solve $1 - 4x + 3x = -1 - 3(2 - x)$.

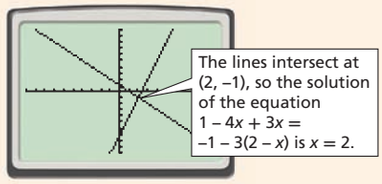
1. Press Y= and enter each side of the equation into Y_1 and Y_2. You do not need to simplify first.



2. Press 2nd TABLE GRAPH to use the **TABLE** function. Scroll until you find the value of x for which the y-values are the same.



3. Press GRAPH and verify where the lines intersect.



4. You can check your answer by solving the equation algebraically.

$$\begin{aligned} 1 - 4x + 3x &= -1 - 3(2 - x) \\ 1 - x &= -1 - 6 + 3x \\ 1 - x - 3x &= -7 + 3x - 3x \\ 1 - 4x &= -7 \\ 1 - 1 - 4x &= -7 - 1 \\ -4x &= -8 \\ \frac{-4x}{-4} &= \frac{-8}{-4} \\ x &= 2 \checkmark \end{aligned}$$

Try This

Solve each equation by graphing.

7. $-5x + 2(x - 2) = 5x + 4$ **–1**
8. $3(x + 2) = -x + 18$ **3**
9. $2x - 5x + 4 = 2(x - 1) + 16$ **–2**

COMMON CORE

1-6 Solving for a Variable

CC.9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.*
Also **CC.9-12.A.REI.3, CC.9-12.N.Q.1***

Objectives
Solve a formula for a given variable.

Solve an equation in two or more variables for one of the variables.

Vocabulary
formula
literal equation

Who uses this?
Athletes can "rearrange" the distance formula to calculate their average speed.

Many wheelchair athletes compete in marathons, which cover about 26.2 miles. Using the time t it took to complete the race, the distance d, and the *formula* $d = rt$, racers can find their average speed r.

A **formula** is an equation that states a rule for a relationship among quantities.

In the formula $d = rt$, d is isolated. You can "rearrange" a formula to isolate any variable by using inverse operations. This is called *solving for a variable.*

Know it! Note

Solving for a Variable	
Step 1	Locate the variable you are asked to solve for in the equation.
Step 2	Identify the operations on this variable and the order in which they are applied.
Step 3	Use inverse operations to undo operations and isolate the variable.

EXAMPLE 1 ***Sports Application***

In 2004, Ernst Van Dyk won the wheelchair race of the Boston Marathon with a time of about 1.3 hours. The race was about 26.2 miles. What was his average speed? Use the formula $d = rt$ and round your answer to the nearest tenth.

The question asks for speed, so first solve the formula $d = rt$ for r.

$d = rt$ *Locate r in the equation.*

$\frac{d}{t} = \frac{rt}{t}$ *Since r is multiplied by t, divide both sides by t to undo the multiplication.*

$\frac{d}{t} = r$, or $r = \frac{d}{t}$

Now use this formula and the information given in the problem.

$r = \frac{d}{t} \approx \frac{26.2}{1.3}$

≈ 20.2

Van Dyk's average speed was about 20.2 miles per hour.

Helpful Hint
A number divided by itself equals 1. For $t \neq 0$, $\frac{t}{t} = 1$.

CHECK IT OUT!

1. Solve the formula $d = rt$ for t. Find the time in hours that it would take Van Dyk to travel 26.2 miles if his average speed was 18 miles per hour. Round to the nearest hundredth.
about 1.46 h

© Ezra Shaw/Getty Images

1-6 Organizer

Pacing: Traditional $1\frac{1}{2}$ days
Block $\frac{3}{4}$ day

Objectives: Solve a formula for a given variable.

Solve an equation in two or more variables for one of the variables.

Online Edition
Tutorial Videos

Countdown

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Warm Up

Solve each equation.

1. $5 + x = -2$ -7

2. $8m = 43$ $\frac{43}{8}$

3. $\frac{c + 5}{4} = 6$ 19

4. $0.3s + 0.6 = 1.5$ 3

5. $10k - 6 = 9k + 2$ 8

Also available online

Math Humor

Q: If you give 15 cents to one friend and 10 cents to another friend, what time is it?

A: A quarter to two.

COMMON CORE

CC.9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.*
CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable…
CC.9-12.N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas….*

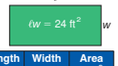
Learn It Online
State Resources Online

1 Introduce

EXPLORATION

Solving for a Variable

Mei is planning a rectangular plot for her vegetable garden. She wants the plot to have an area of 24 ft².



Length ℓ (ft)	Width w (ft)	Area A (ft²)
1	24	24
2	12	24
3	8	24
4	6	24
6	4	24
8	3	24

1. Mei considers various lengths for the plot. Complete the table to show the corresponding width for each length.
2. Explain how you found each width. Divide 24 by the length.
3. Write an equation that gives the width w of the plot for any length ℓ. $w = \frac{24}{\ell}$

THINK AND DISCUSS

4. **Explain** how you can use your equation to find the width of the plot when the length is 15 feet.
5. **Describe** how the equation you wrote in Problem 3 is related to the original area equation $\ell w = 24$.

4. Substitute 15 for ℓ in the equation: $w = \frac{24}{15} = 1.6$ ft.
5. If you divide both sides of the orig. area equation by ℓ, the result is the equation from Problem 3.

Motivate

Show students these two equivalent equations that relate the area of a rectangle to its length and width: $A = \ell w$ and $\ell = \frac{A}{w}$. Explain that the first equation is more convenient if you know the length and the width and want to find the area. The second equation is more convenient if you know the area and the width and want to find the length.

Explorations and answers are provided in your online resources.

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Additional Examples

Example 1

The formula $C = \pi d$ gives the circumference of a circle C in terms of its diameter d. The circumference of a bowl is 18 inches. What is the bowl's diameter? Leave the symbol π in your answer.

$\frac{18}{\pi}$ in.

Example 2

A. The formula for the area of a triangle is $A = \frac{1}{2}bh$, where b is the length of the base and h is the height. Solve for h.

$h = \frac{2A}{b}$

B. Solve the formula for a person's typing speed for e.

$e = \frac{ms - w}{-10}$

Example 3

A. Solve $x + y = 15$ for x.

$x = 15 - y$

B. Solve $pq = x$ for q. $q = \frac{x}{p}$

Also available online

Mathematical Practices **Questioning Strategies**

EXAMPLES 1–3

- How is solving for a variable similar to solving an equation? How is it different?

EXAMPLE 2 Solving Formulas for a Variable

A The formula for a Fahrenheit temperature in terms of degrees Celsius is $F = \frac{9}{5}C + 32$. Solve for C.

$F = \frac{9}{5}C + 32$ — Locate C in the equation.

$-32 \quad -32$ — Since 32 is added to $\frac{9}{5}C$, subtract 32 from both sides to undo the addition.

$F - 32 = \frac{9}{5}C$

$\left(\frac{5}{9}\right)(F - 32) = \left(\frac{5}{9}\right)\frac{9}{5}C$ — Since C is multiplied by $\frac{9}{5}$, divide both sides by $\frac{9}{5}$ (multiply by $\frac{5}{9}$) to undo the multiplication.

$\frac{5}{9}(F - 32) = C$

Remember!

Dividing by a fraction is the same as multiplying by the reciprocal.

B The formula for a person's typing speed is $s = \frac{w - 10e}{m}$, where s is speed in words per minute, w is number of words typed, e is number of errors, and m is number of minutes typing. Solve for w.

$s = \frac{w - 10e}{m}$ — Locate w in the equation.

$m(s) = m\left(\frac{w - 10e}{m}\right)$ — Since w − 10e is divided by m, multiply both sides by m to undo the division.

$ms = w - 10e$

$+10e \quad +10e$ — Since 10e is subtracted from w, add 10e to both sides to undo the subtraction.

$ms + 10e = w$

CHECK IT OUT! **2.** The formula for an object's final velocity f is $f = i - gt$, where i is the object's initial velocity, g is acceleration due to gravity, and t is time. Solve for i. $i = f + gt$

A formula is a type of *literal equation*. A **literal equation** is an equation with two or more variables. To solve for one of the variables, use inverse operations.

EXAMPLE 3 Solving Literal Equations for a Variable

A Solve $m - n = 5$ for m.

$m - n = 5$ — Locate m in the equation.

$+n \quad +n$ — Since n is subtracted from m, add n to both sides to undo the subtraction.

$m = 5 + n$

B Solve $\frac{m}{k} = x$ for k.

$\frac{m}{k} = x$ — Locate k in the equation.

$k\left(\frac{m}{k}\right) = kx$ — Since k appears in the denominator, multiply both sides by k.

$m = kx$

$\frac{m}{x} = \frac{kx}{x}$ — Since k is multiplied by x, divide both sides by x to undo the multiplication.

$\frac{m}{x} = k$

CHECK IT OUT! **3a.** Solve $5 - b = 2t$ for t. $t = \frac{5 - b}{2}$

3b. Solve $D = \frac{m}{V}$ for V. $V = \frac{m}{D}$

2 Teach

Guided Instruction

Solve $x + 7 = 9$ and $\frac{x}{7} = 9$. Then replace the 7 with y and solve for x again. Show that the procedures are the same, but the answers are in a different form.

Teaching Tip **Visual** Have students circle the variable for which they are solving. On the board, circle this variable as well, or write the variable in a different color.

3 Close

Summarize

Ask students to identify the first step in solving each of the following literal equations for x.

$abx = c$ Divide both sides by ab.

$ax + b = c$ Subtract b from both sides.

$\frac{x - a}{b} = c$ Multiply both sides by b.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

THINK AND DISCUSS

1. Describe a situation in which a formula could be used more easily if it were "rearranged." Include the formula in your description.
2. Explain how to solve $P = 2\ell + 2w$ for w.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Write a formula that is used in each subject. Then solve the formula for each of its variables.

Common Formulas	
Subject	**Formula**
Geometry	
Physical science	
Earth science	

1-6 Exercises

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GUIDED PRACTICE

1. **Vocabulary** Explain why a *formula* is a type of *literal equation*.

SEE EXAMPLE 1

2. **Construction** The formula $a = 46c$ gives the floor area a in square meters that can be wired using c circuits.
   a. Solve $a = 46c$ for c. $c = \frac{a}{46}$
   b. If a room is 322 square meters, how many circuits are required to wire this room? 7 circuits

SEE EXAMPLE 2

3. The formula for the volume of a rectangular prism with length ℓ, width w, and height h is $V = \ell wh$. Solve this formula for w.

SEE EXAMPLE 3

4. Solve $st + 3t = 6$ for s. $s = \frac{6 - 3t}{t}$
5. Solve $m - 4n = 8$ for m. $m = 4n + 8$
6. Solve $\frac{f+4}{g} = 6$ for f. $f = 6g - 4$
7. Solve $b + c = \frac{10}{a}$ for a. $a = \frac{10}{b + c}$

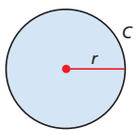
PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
8	1
9	2
10–13	3

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

8. **Geometry** The formula $C = 2\pi r$ relates the circumference C of a circle to its radius r. (Recall that π is the constant ratio of circumference to diameter.)
   a. Solve $C = 2\pi r$ for r. $r = \frac{C}{2\pi}$
   b. If a circle's circumference is 15 inches, what is its radius? Leave the symbol π in your answer. $\frac{7.5}{\pi}$ in.



C is the distance around the circle.
r is the distance from the center of the circle to any point on the circle.

9. **Finance** The formula $A = P + I$ shows that the total amount of money A received from an investment equals the principal P (the original amount of money invested) plus the interest I. Solve this formula for I. $I = A - P$
10. Solve $-2 = 4r + s$ for s. $s = -2 - 4r$
11. Solve $xy - 5 = k$ for x. $x = \frac{k+5}{y}$
12. Solve $\frac{m}{n} = p - 6$ for n. $\frac{m}{p-6} = n$
13. Solve $\frac{x-2}{y} = z$ for y. $\frac{x-2}{z} = y$

Answers

1. A literal equation contains more than one variable. A formula shows how to determine the value of one variable when you know the value(s) of one or more other variables. So a formula always contains more than one variable, making it a literal equation.
3. $w = \frac{V}{\ell h}$

Answers to *Think and Discuss*

1. Possible answer: The formula $d = rt$ is more useful in the form $\frac{d}{r} = t$ when you need to determine the time it takes to travel a certain distance at a certain speed.
2. Isolate w on the right side of the equation by subtracting 2ℓ from both sides and then dividing both sides by 2.
3. See Additional Answers.

1-6 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–3

Basic	8–31, 34–37
Average	8–13, 20–41
Advanced	8–13, 20–28 even, 29–45

Homework Quick Check
Quickly check key concepts.
Exercises: 8, 9, 10, 12, 24, 28, 30

Make sense of problems and persevere in solving them. Exercises 2, 8–9, 30–31, 34, 45

Reason abstractly and quantitatively. Exercise 29

Construct viable arguments and critique the reasoning of others. Exercises 32–33

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Teaching Tip **Diversity** **Exercise 30** is about baseball and may use terms unfamiliar to some students. Have students familiar with baseball discuss the meanings of *run*, *earned run*, *inning*, etc.

Multi-Step Test Prep **Exercise 34** involves using a known rate to find data values and then displaying that data visually. This exercise prepares students for the Multi-Step Test Prep.

Answers

14. $n = \frac{S + 360}{180}$
15. $x = 5(a + g)$
16. $b = \frac{2A}{h}$
17. $x = \frac{y - b}{m}$
18. $n = \frac{a - 1}{3}$
19. $T = \frac{PV}{nR}$
20. $T = R - M$
21. $T = M + R$
22. $R = \frac{PV}{nT}$
23. $b = \frac{c - 2a}{2}$
24. $c = \frac{-4p}{9}$
25. $r = 7 - ax$
26. $y = \frac{2 - 3x}{7}$
27. $x = \frac{5 - 4y}{3}$
28. $b = \frac{y - 3x}{3}$
32. Possible answer: Use inverse operations to isolate the indicated variable on one side of the equation. Be sure the variable is the only expression on one side of the equation and doesn't appear on the other side.
34c. For graph, See Additional Answers.

Solve for the indicated variable.

14. $S = 180n - 360$ for n
15. $\frac{x}{5} - g = a$ for x
16. $A = \frac{1}{2}bh$ for b
17. $y = mx + b$ for x
18. $a = 3n + 1$ for n
19. $PV = nRT$ for T
20. $T + M = R$ for T
21. $M = T - R$ for T
22. $PV = nRT$ for R
23. $2a + 2b = c$ for b
24. $5p + 9c = p$ for c
25. $ax + r = 7$ for r
26. $3x + 7y = 2$ for y
27. $4y + 3x = 5$ for x
28. $y = 3x + 3b$ for b

29. **Estimation** The table shows the flying time and distance traveled for five flights on a certain airplane.

Flying Times		
Flight	Time (h)	Distance (mi)
A	2	1018
B	3	1485
C	4	2103
D	5	2516
E	6	2886

29a. Possible answer: $t \approx \frac{d}{500}$

a. Use the data in the table to write a rule that *estimates* the relationship between flying time t and distance traveled d.

b. Possible answer: about 2.6 h

b. Use your rule from part **a** to estimate the time that it takes the airplane to fly 1300 miles.

c. Possible answer: $d \approx 500t$

c. Solve your rule for d.

d. Possible answer: about 4000 mi

d. Use your rule from part **c** to estimate the distance the airplane can fly in 8 hours.

30. **Sports** To find a baseball pitcher's earned run average (ERA), you can use the formula $Ei = 9r$, where E represents ERA, i represents number of innings pitched, and r represents number of earned runs allowed. Solve the equation for E. What is a pitcher's ERA if he allows 5 earned runs in 18 innings pitched? $E = \frac{9r}{i}$; 2.5

31. $a = \frac{t - g}{-0.0035}$

31. **Meteorology** For altitudes up to 36,000 feet, the relationship between temperature and altitude can be described by the formula $t = -0.0035a + g$, where t is the temperature in degrees Fahrenheit, a is the altitude in feet, and g is the ground temperature in degrees Fahrenheit. Solve this formula for a.

32. **Write About It** In your own words, explain how to solve a literal equation for one of the variables.

33. Possible answer: The variable a appears in 2 terms. Use the Distributive Property to write $a - ab$ as $a(1 - b)$, and then divide both sides by $1 - b$.

33. **Critical Thinking** How is solving $a - ab = c$ for a different from the problems in this lesson? How might you solve this equation for a?

Multi-Step Test Prep

34. **a.** Suppose firefighters can extinguish a wildfire at a rate of 60 acres per day. Use this information to complete the table.

Days	Acres
1	60
2	120
3	180
4	240
5	300
d	$60d$

b. $A = 60d$

b. Use the last row in the table to write an equation for acres A extinguished in terms of the number of days d.

c. Graph the points in the table with *Days* on the horizontal axis and *Acres* on the vertical axis. Describe the graph. The graph is a line.

(br) ©Index Stock/Alamy; (cr) Afo Sport

PRACTICE A
PRACTICE C
PRACTICE B

Answer each of the following.

1. The formula $C = 2\pi r$ relates the radius r of a circle to its circumference C. Solve the formula for r. $r = \frac{C}{2\pi}$
2. The formula $y = mx + b$ is called the slope-intercept form of a line. Solve this formula for m. $m = \frac{y - b}{x}$

Solve for the indicated variable.

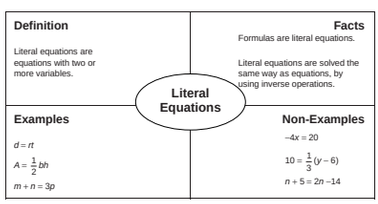
3. $4c = d$ for c $c = \frac{d}{4}$
4. $n - 6m = 8$ for n $n = 8 + 6m$
5. $2p + 5r = q$ for p $p = \frac{q - 5r}{2}$
6. $-10 = xy + z$ for x $x = \frac{-10 - z}{y}$
7. $\frac{a}{b} = c$ for b $b = \frac{a}{c}$
8. $\frac{h - 4}{j} = k$ for j $j = \frac{h - 4}{k}$

Answer each of the following.

9. The formula $c = 5p + 215$ relates c, the total cost in dollars of hosting a birthday party at a skating rink, to p, the number of people attending.
   a. Solve the formula $c = 5p + 215$ for p. $p = \frac{c - 215}{5}$
   b. If Allie's parents are willing to spend $300 for a party, how many people can attend? 17
10. The formula for the area of a triangle is $A = \frac{1}{2}bh$, where b represents the length of the base and h represents the height.
    a. Solve the formula $A = \frac{1}{2}bh$ for b. $b = \frac{2A}{h}$
    b. If a triangle has an area of 192 mm^2, and the height measures 12 mm, what is the measure of the base? 32 mm

READING STRATEGIES

Use the concept map below to help you understand literal equations.



Answer each question.

1. Give your own example of a literal equation. Possible answer: $3x + 2y = 9$
2. Why is $n + 5 = 2n - 14$ given as a non-example? The equation contains only one variable, n.
3. Is an equation with four different variables a literal equation? Why? Yes, because it has two or more variables.
4. Describe how to solve $d = rt$ for t. Divide both sides by r.
5. Solve the literal equation $3t + 8 = b$ for t. $t = \frac{b - 8}{3}$
6. The formula for the volume of a rectangular prism is $V = lwh$.
   a. Solve this formula for h. $h = \frac{V}{lw}$
   b. Find the height of a rectangular prism with a volume of 189 cm^2, a length of 9 cm, and a width of 7 cm. 3 cm

RETEACH

Solving for a variable in a formula can make it easier to use that formula. The process is similar to that of solving multi-step equations. Find the operations being performed on the variable you are solving for, and then use inverse operations.

	Operations	Solve using Inverse Operations
$A = lw$ Solve for w.	• w is multiplied by l.	• Divide both sides by l.
$P = 2l + 2w$ Solve for w.	• w is multiplied by 2. • Then $2l$ is added.	• Add $-2l$ to both sides. • Then divide both sides by 2.

The order of the inverse operations is the order of operations in reverse.

The formula $A = \frac{1}{2}bh$ relates the area A of a triangle to its base b and height h. Solve the formula for b.

$A = \frac{1}{2}bh$	b is multiplied by $\frac{1}{2}$.
$\left(\frac{2}{1}\right) \cdot A = \left(\frac{2}{1}\right)\frac{1}{2}bh$	Multiply both sides by $\frac{2}{1}$.
$2A = bh$	b is multiplied by h.
$\frac{2A}{h} = \frac{bh}{h}$	Divide both sides by h.
$\frac{2A}{h} = b$	Simplify.

Solve for the indicated variable.

1. $P = 4s$ for s $s = \frac{P}{4}$
2. $a + b + c = 180$ for b $b = 180 - a - c$
3. $P = \frac{KT}{V}$ for K $K = \frac{VP}{T}$

The formula $V = \frac{1}{3}lwh$ relates the volume of a square pyramid to its base length l, base width w, and height h.

4. Solve the formula for w. $w = \frac{3V}{lh}$
5. A square pyramid has a volume of 560 in^3, a base length of 10 in., and a height of 14 in. What is its base width? 12 in.

TEST PREP

35. Which equation is the result of solving $9 + 3x = 2y$ for x?

Ⓐ $\frac{9 + 3y}{2} = x$ Ⓑ $\frac{2}{3}y - 9 = x$ Ⓒ $x = \frac{2}{3}y - 3$ Ⓓ $x = 2y - 3$

36. Which of the following is a correct method for solving $2a - 5b = 10$ for b?

Ⓕ Add $5b$ to both sides, then divide both sides by 2.

Ⓖ Subtract $5b$ from both sides, then divide both sides by 2.

Ⓗ Divide both sides by 5, then add $2a$ to both sides.

Ⓙ Subtract $2a$ from both sides, then divide both sides by -5.

37. The formula for the volume of a rectangular prism is $V = \ell wh$. Anna wants to make a cardboard box with a length of 7 inches, a width of 5 inches, and a volume of 210 cubic inches. Which variable does Anna need to solve for in order to build her box?

Ⓐ V Ⓑ ℓ Ⓒ w Ⓓ h

CHALLENGE AND EXTEND

Solve for the indicated variable.

38. $3.3x + r = 23.1$ for x **39.** $\frac{2}{5}a - \frac{3}{4}b = c$ for a **40.** $\frac{3}{5}x + 1.4y = \frac{2}{5}$ for y

41. $t = \frac{d}{500} + \frac{1}{2}$ for d **42.** $s = \frac{1}{2}gt^2$ for g **43.** $v^2 = u^2 + 2as$ for s

44. Solve $y = mx + 6$ for m. What can you say about y if $m = 0$?

45. Entertainment The formula $S = \frac{h \cdot w \cdot f \cdot t}{35{,}000}$ gives the approximate size in kilobytes (Kb) of a compressed video. The variables h and w represent the height and width of the frame measured in pixels, f is the number of frames per second (fps) the video plays, and t is the time the video plays in seconds. Estimate the time a movie trailer will play if it has a frame height of 320 pixels, has a frame width of 144 pixels, plays at 15 fps, and has a size of 2370 Kb. **120 s**

© Pixar Animation Studios/ZUMA Press

38. $x = \frac{23.1 - r}{3.3}$

39. $a = \frac{5}{2}\left(c + \frac{3}{4}b\right)$

40. $y = \frac{\frac{2}{5} - \frac{3}{5}x}{1.4}$

41. $d = 500\left(t - \frac{1}{2}\right)$

42. $g = \frac{2s}{t^2}$

43. $s = \frac{v^2 - u^2}{2a}$

44. $m = \frac{y - 6}{x}$; if $m = 0$, $y = 6$

PROBLEM SOLVING

Use the table below, which shows some track and field gold medal winners, to answer questions 1–4. Round all answers to the nearest tenth.

2000 Summer Olympics		
Gold Medal Winner	**Race**	**Time (s)**
M. Greene, USA	100 m	9.87
K. Kenteris, Greece	200 m	20.09
M. Johnson, USA	400 m	43.84
A. Garcia, Cuba	110 m hurdles	13.00

1. Solve the formula $d = rt$ for r. $r = \frac{d}{t}$
2. Find Johnson's average speed in meters per second. **9.1 m/s**
3. Find Garcia's average speed in meters per second. **8.5 m/s**
4. The world record of 19.32 seconds in the 200-meter race was set by Michael Johnson in 1996. Find the difference between Johnson's average speed and Kenteris' average speed. **0.4 m/s**

Select the best answer.

5. The cost to mail a letter in the United States in 2008 was $0.41 for the first ounce and $0.26 for each additional ounce. Solve $C = 0.41 + 0.26(z - 1)$ for z.
   A $z = \frac{C - 0.41}{0.26}$
   B $z = \frac{C - 0.41}{0.26} + 1$
   C $z = \frac{C + 0.15}{0.26}$
   D $z = C - 0.67$
6. The formula $V = \frac{Bh}{3}$ shows how to find the volume of a pyramid. Solve for B.
   F $B = \frac{3V}{h}$ H $B = 3Vh$
   G $B = 3V - h$ J $B = 3V + h$
7. Degrees Celsius and degrees Fahrenheit are related by the equation $C = \frac{5}{9}(F - 32)$. Solve for F.
   A $F = 9C + 27$ C $F = \frac{5}{9}C + 32$
   B $F = \frac{9}{5}C$ D $F = \frac{9}{5}C + 32$
8. The cost of operating an electrical device is given by the formula $C = \frac{Wtc}{1000}$ where W is the power in watts, t is the time in hours, and c is the cost in cents per kilowatt-hour. Solve for W.
   F $W = 1000C - tc$
   G $W = \frac{Ctc}{1000}$
   H $W = 1000C + tc$
   J $W = \frac{1000C}{tc}$

CHALLENGE

When you put your money in a savings account, the bank may pay you *simple interest*. Let P represent the dollar amount of your deposit (the principal), let r represent the interest rate, and let t represent the number of years. The amount of interest you earn, I, is given by the *simple interest formula*: $I = Prt$.

Note that banks typically use percents to describe their interest rates. Percent means "per hundred," so an interest rate of 5% means that you should use $r = \frac{5}{100}$, or 0.05.

Use the simple interest formula to solve the following problems:

1. If $P = 2500$, $r = 0.03$, and $t = 5$, what is I? **$I = 375$**
2. If $r = 0.025$, $t = 3$, and $I = 150$, what is P? **$P = 2000$**
3. If $P = 500$, $r = 0.06$, and $I = 150$, what is t? **$t = 5$ years**
4. If $P = 3000$, $t = 4$, and $I = 384$, what is r? **$r = 0.032$ or 3.2%**
5. Kevin is making a deposit of $1800 at his local bank. The bank pays 6.5% simple interest ($r = 0.065$). If Kevin leaves his deposit at the bank for 3 years, how much interest will he earn? **$351**
6. Cecelia made a deposit of $600 at a bank paying 4% simple interest ($r = 0.04$). How long should she leave her deposit at the bank in order to earn $72 in interest? **3 years**
7. Darryl opened an account at a bank which paid 5.5% simple interest ($r = 0.055$). After 6 years, he had earned $726 in interest. What was the amount of his original deposit? **$2200**
8. Sophia deposited $150 at a savings and loan association paying simple interest. If she earned $27 in interest after 6 years, what was the interest rate? **0.03 or 3%**
9. Nathan made a deposit of $650 at a bank paying 3.8% simple interest ($r = 0.038$). If he leaves his deposit at the bank for 10 years, how much interest will he earn? **$247**
10. Susie made a deposit of $980 at a credit union paying 7% simple interest ($r = 0.07$). How long should she leave her deposit at the credit union in order to earn $343 in interest? **5 years**
11. Guillermo deposited $1350 at a bank paying simple interest. If he earned $109.35 in 3 years, what was the interest rate? **0.027 or 2.7%**

COMMON ERROR ALERT

Students may subtract when they should divide. In **Exercise 30,** for example, they may write $r = 9E - i$. Ask students which operation is being performed on r and what will "undo" it.

TEST PREP DOCTOR

In **Exercise 35,** students who chose **B** or **D** most likely did not divide every term by 3.

Students who chose **F** in **Exercise 36** probably solved the equation for a. Encourage students to read the question carefully.

Journal

Have students discuss the differences in solving the following literal equations for x: $\frac{x}{7} - y = t$ and $\frac{x - y}{7} = t$.

ALTERNATIVE ASSESSMENT

Have students explain how to solve for each of the three variables in the following literal equation: $q + r = \frac{p}{5}$.

Power Presentations with PowerPoint®

1-6 Lesson Quiz

Solve for the indicated variable.

1. $V = \frac{1}{3}Ah$ for h $h = \frac{3V}{A}$
2. $P = R - C$ for C $C = R - P$
3. $2x + 7y = 14$ for y $y = \frac{14 - 2x}{7}$
4. $\frac{m}{x} = k - 6$ for m $m = x(k - 6)$
5. $R = \frac{C - S}{t}$ for C $C = Rt + S$

Euler's formula, $V - E + F = 2$, relates the number of vertices V, the number of edges E, and the number of faces F of a polyhedron.

6. Solve Euler's formula for F. $F = 2 - V + E$
7. How many faces does a polyhedron with 8 vertices and 12 edges have? 6

Also available online

1-7 Organizer

Pacing: Traditional $1\frac{1}{2}$ days
Block $\frac{3}{4}$ day

Objectives: Solve equations in one variable that contain absolute-value expressions.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Solve.

1. $x - 10 = 4$ 14
2. $s + 5 = -2$ −7
3. $32 = -8y$ −4
4. $\frac{m+4}{2} = 7$ 10
5. $-14 = x - 5$ −9
6. $2t + 5 = 45$ 20

Also available online

Math Humor

Question: How is the equation $|x| = 8$ similar to a chemist's laboratory?

Answer: Both have multiple solutions.

COMMON CORE

CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems.*
CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

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COMMON CORE

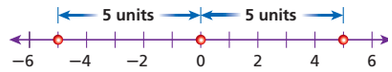
1-7 Solving Absolute-Value Equations

CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems.* *Also* **CC.9-12.A.REI.3**

Objective
Solve equations in one variable that contain absolute-value expressions.

Why learn this?
Engineers can solve absolute-value equations to calculate the length of the deck of a bridge. (See Example 3.)

Recall that the absolute value of a number is that number's distance from zero on a number line. For example, $|-5| = 5$ and $|5| = 5$.

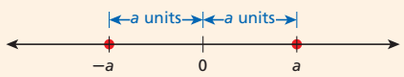


For any nonzero absolute value, there are exactly two numbers with that absolute value. For example, both 5 and −5 have an absolute value of 5.

To write this statement using algebra, you would write $|x| = 5$. This equation asks, "What values of x have an absolute value of 5?" The solutions are 5 and −5. Notice that this equation has two solutions.

Know it! Note

Absolute-Value Equations

WORDS	NUMBERS
The equation $\|x\| = a$ asks, "What values of x have an absolute value of a?" The solutions are a and the opposite of a.	$\|x\| = 5$ $x = 5$ or $x = -5$
GRAPH	**ALGEBRA**
	$\|x\| = a$ $x = a$ or $x = -a$ $(a \geq 0)$

To solve absolute-value equations, perform inverse operations to isolate the absolute-value expression on one side of the equation. Then you must consider two cases.

EXAMPLE 1 **Solving Absolute-Value Equations**

Solve each equation.

A $|x| = 4$

$|x| = 4$ *Think: What numbers are 4 units from 0?*

Case 1	Case 2
$x = -4$	$x = 4$

Rewrite the equation as two cases.

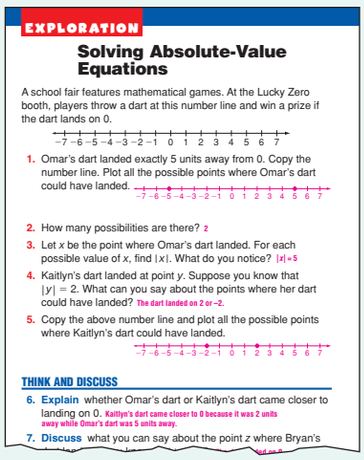
The solutions are −4 and 4.

Helpful Hint

Be sure to check both solutions when you solve an absolute-value equation.

$\|x\| = 4$	$\|x\| = 4$
$\|-4\|$ | 4	$\|4\|$ | 4
4 | 4 ✓	4 | 4 ✓

1 Introduce



EXPLORATION

Solving Absolute-Value Equations

A school fair features mathematical games. At the Lucky Zero booth, players throw a dart at this number line and win a prize if the dart lands on 0.

1. Omar's dart landed exactly 5 units away from 0. Copy the number line. Plot all the possible points where Omar's dart could have landed.
2. How many possibilities are there? 2
3. Let x be the point where Omar's dart landed. For each possible value of x, find $|x|$. What do you notice? $|x| = 5$
4. Kaitlyn's dart landed at point y. Suppose you know that $|y| = 2$. What can you say about the points where her dart could have landed? The dart landed on 2 or −2.
5. Copy the above number line and plot all the possible points where Kaitlyn's dart could have landed.

THINK AND DISCUSS

6. **Explain** whether Omar's dart or Kaitlyn's dart came closer to landing on 0. Kaitlyn's dart came closer to 0 because it was 2 units away while Omar's dart was 5 units away.
7. **Discuss** what you can say about the point z where Bryan's

Motivate

Present the following situation to the class. In a game show, two contestants guess the price of a DVD player. The contestants each make different guesses, and both differ from the actual price by $15. The actual price of the DVD player is $169. What guesses do the contestants make? 154, 184

Tell students they will learn how to use absolute-value equations to solve problems like this.

Explorations and answers are provided in your online resources.

Solve each equation.

B $4|x+2| = 24$

$\frac{4|x+2|}{4} = \frac{24}{4}$ *Since $|x + 2|$ is multiplied by 4, divide both sides by 4 to undo the multiplication.*

$|x+2| = 6$ *Think: What numbers are 6 units from 0?*

Case 1	Case 2
$x + 2 = -6$	$x + 2 = 6$
$-2 \quad -2$	$-2 \quad -2$
$x = -8$	$x = 4$

Rewrite the equation as two cases. Since 2 is added to x, subtract 2 from both sides of the equation.

The solutions are –8 and 4.

CHECK IT OUT! **Solve each equation. Check your answer.**

1a. $|x| - 3 = 4$ **–7, 7** **1b.** $8 = |x - 2.5|$ **–5.5, 10.5**

The table summarizes the steps for solving absolute-value equations.

Know it! Note

Solving an Absolute-Value Equation
1. Use inverse operations to isolate the absolute-value expression.
2. Rewrite the resulting equation as two cases that do not involve absolute values.
3. Solve the equation in each of the two cases.

Not all absolute-value equations have two solutions. If the absolute-value expression equals 0, there is one solution. If an equation states that an absolute value is negative, there are no solutions.

EXAMPLE 2 Special Cases of Absolute-Value Equations

Solve each equation.

A $|x+3|+4 = 4$

$|x+3|+4 = 4$ *Since 4 is added to $|x + 3|$, subtract 4 from both sides to undo the addition.*

$-4 \quad -4$

$|x+3| = 0$

$x + 3 = 0$ *There is only one case. Since 3 is added to x, subtract 3 from both sides to undo the addition.*

$-3 \quad -3$

$x = -3$

B $5 = |x+2|+8$

$5 = |x+2|+8$ *Since 8 is added to $|x + 2|$, subtract 8 from both sides to undo the addition.*

$-8 \quad -8$

$-3 = |x+2|$ ✗ *Absolute value cannot be negative.*

This equation has no solution.

Remember!
Absolute value must be nonnegative because it represents a distance.

CHECK IT OUT! **Solve each equation.**

2a. $2 - |2x - 5| = 7$ **no solution** **2b.** $-6 + |x - 4| = -6$ **4**

2 Teach

Guided Instruction

Review with students the concept of absolute value. Remind them that the absolute value of a number represents its distance from 0. Because distance can be in the positive or negative direction, there can be as many as two possible solutions for an absolute-value equation. Because distance is always nonnegative, there is no solution when an absolute-value expression is equal to a negative number.

Reaching All Learners

Through Concrete Manipulatives

To help students focus on first isolating the absolute-value expression, have them place a small object or piece of paper over the absolute-value expression in an equation. Then have them "isolate" the object or piece of paper by using inverse operations. Lastly, have students remove the small object or piece of paper and rewrite the equation with two cases.

COMMON ERROR ALERT

In **Example 1B,** students may try to distribute 4 over the absolute value bars. Remind students that their first step should be to isolate the absolute-value expression by dividing both sides by the number in front of the absolute-value bars.

Power Presentations with PowerPoint®

Additional Examples

Example 1

Solve each equation.

A. $|x| = 12$ $-12, 12$

B. $3|x + 7| = 24$ $-15, 1$

Example 2

Solve each equation.

A. $-8 = |x + 2| - 8$ -2

B. $3 + |x + 4| = 0$ no solution

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- What does *absolute value* mean?
- How is the absolute-value sign similar to parentheses? How is it different?
- Why must you consider two cases when solving absolute-value equations?

EXAMPLE 2

- How can you tell when an absolute-value equation has one solution?
- How can you tell when an absolute-value equation has no solution?

Teaching Tip **Auditory** When reading absolute-value expressions aloud, students often confuse expressions of the type $|x + 4|$ and $|x| + 4$. Be sure they read the first equation as "the absolute value of the quantity *x* plus four" and the second equation as "the absolute value of *x* plus four."

Power Presentations with PowerPoint®

Additional Examples

Example 3

A support beam for a building must be 3.5 meters long. It is acceptable for the beam to differ from the ideal length by 3 millimeters. Write and solve an absolute-value equation to find the minimum and maximum acceptable lengths for the beam. $|x - 3.5| = 0.003$; 3.497 m; 3.503 m

Also available online

Questioning Strategies

EXAMPLE 3

- What is the relationship between millimeters and meters?
- What must be true about the minimum and maximum lengths when you plot these values on a number line?

Answers to *Think and Discuss*

1. Divide both sides by $\frac{1}{5}$ (or multiply both sides by 5). Then write two cases: $x - 3 = -10$ and $x - 3 = 10$. Solve these equations to get $x = -7$ or $x = 13$.
2. See Additional Answers.

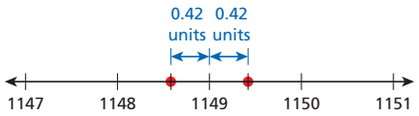
EXAMPLE 3 ***Engineering Application***

Sydney Harbour Bridge in Australia is 1149 meters long. Because of changes in temperature, the bridge can expand or contract by as much as 420 millimeters. Write and solve an absolute-value equation to find the minimum and maximum lengths of the bridge.

First convert millimeters to meters.

420 mm = 0.4 2 0 m *Move the decimal point three places to the left.*

The length of the bridge can vary by 0.42 m, so find two numbers that are 0.42 units away from 1149 on a number line.



You can find these numbers by using the absolute-value equation $|x - 1149| = 0.42$. Solve the equation by rewriting it as two cases.

Case 1			Case 2			
$x - 1149 =$	-0.42		$x - 1149 =$	0.42		*Since 1149 is subtracted from x, add 1149 to both sides of each equation.*
$+ 1149$	$+ 1149$		$+ 1149$	$+ 1149$		
$x =$	1148.58		$x =$	1149.42		

The minimum length of the bridge is 1148.58 m, and the maximum length is 1149.42 m.

CHECK IT OUT!

3. Sydney Harbour Bridge is 134 meters tall. The height of the bridge can rise or fall by 180 millimeters because of changes in temperature. Write and solve an absolute-value equation to find the minimum and maximum heights of the bridge.

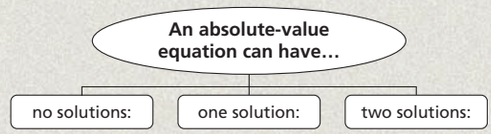
3. $|x - 134| = 0.18$; min. height: 133.82 m; max. height: 134.18 m

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Explain the steps you would use to solve the equation $\frac{1}{5}|x - 3| = 2$.

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of an absolute-value equation that has the indicated number of solutions, and then solve.



Know it! Note

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3 Close

Summarize

Ask students to summarize the steps in solving an absolute-value equation.

1. Isolate the absolute-value expression.
2. Rewrite the equation as two cases.
3. Solve for the variable in both cases.
4. Check.

Then ask, "When will there be two solutions? one solution? no solution?" When the expression equals a positive number, equals zero, and equals a negative number, respectively.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

1-7 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

Solve each equation.

SEE EXAMPLE 1

1. $|x| = 6$ **−6, 6**
2. $9 = |x + 5|$ **−14, 4**
3. $|3x| + 2 = 8$ **−2, 2**
4. $2|x| = 18$ **−9, 9**
5. $\left|x + \frac{1}{2}\right| = 1$ **$-\frac{3}{2}, \frac{1}{2}$**
6. $|x - 3| - 6 = 2$ **−5, 11**

SEE EXAMPLE 2

7. $-8 = |x|$ **no solution**
8. $|x| = 0$ **0**
9. $|x + 4| = -7$ **no solution**
10. $7 = |3x + 9| + 7$ **−3**
11. $|2.8 - x| + 1.5 = 1.5$ **2.8**
12. $5|x + 7| + 14 = 8$ **no solution**

SEE EXAMPLE 3

13. **Communication** Barry's walkie-talkie has a range of 2 mi. Barry is traveling on a straight highway and is at mile marker 207. Write and solve an absolute-value equation to find the minimum and maximum mile marker from 207 that Barry's walkie-talkie will reach. **$|x - 207| = 2$; mile markers 205 and 209**

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–22	1
23–28	2
29	3

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

Solve each equation.

14. $|x| = \frac{1}{5}$ **$-\frac{1}{5}, \frac{1}{5}$**
15. $|2x - 4| = 22$ **−9, 13**
16. $18 = 3|x - 1|$ **−5, 7**
17. $-2|x| = -4$ **−2, 2**
18. $3|x| - 12 = 18$ **−10, 10**
19. $|x - 42.04| = 23.24$ **18.8, 65.28**
20. $\left|\frac{2}{3}x - \frac{2}{3}\right| = \frac{2}{3}$ **0, 2**
21. $|3x + 1| = 13$ **$-\frac{14}{3}$, 4**
22. $|-2x + 3| = 5.8$ **−1.4, 4.4**
23. $|4x| + 9 = 9$ **0**
24. $8 = 7 - |x|$ **no solution**
25. $|x| + 6 = 12 - 6$ **0**
26. $|x - 3| + 14 = 5$ **no solution**
27. $0 = \left|\frac{2}{3} - x\right|$ **$\frac{2}{3}$**
28. $3 + |x - 1| = 3$ **1**

29. **Space Shuttle** The diameter of a valve for the space shuttle must be within 0.001 mm of 5 mm. Write and solve an absolute-value equation to find the boundary values for the acceptable diameters of the valve.

30. The two numbers that are 5 units from 3 on the number line are represented by the equation $|n - 3| = 5$. What are these two numbers? Graph the solutions. **8, −2**

31. Write and solve an absolute-value equation that represents two numbers x that are 2 units from 7 on a number line. Graph the solutions.

32. **Manufacturing** A quality control inspector at a bolt factory examines random bolts that come off the assembly line. Any bolt whose diameter differs by more than 0.04 mm from 6.5 mm is sent back. Write and solve an absolute-value equation to find the maximum and minimum diameters of an acceptable bolt.

33. **Construction** A brick company guarantees to fill a contractor's order to within 5% accuracy. A contractor orders 1500 bricks. Write and solve an absolute-value equation to find the maximum and minimum number of bricks guaranteed.

34. **Multi-Step** A machine prints posters and then trims them to the correct size. The equation $|\ell - 65.1| = 0.2$ gives the maximum and minimum acceptable lengths for the posters in inches. Does a poster with a length of 64.8 inches fall within the acceptable range? Why or why not? **No; the acceptable range is from 64.9 to 65.3.**

29. $|x - 5| = 0.001$; 4.999 mm; 5.001 mm

31. $|x - 7| = 2$; 5, 9

32. $|x - 6.5| = 0.04$; 6.54 mm; 6.46 mm

33. $|x - 1500| = 75$; 1575 bricks; 1425 bricks

1-7 Exercises

Assignment Guide

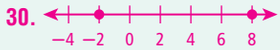
Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–3**

Basic 14–29, 32, 33, 35–38, 45–49
Average 14–29, 32–50
Advanced 14–28 even, 29, 32–52

Homework Quick Check
Quickly check key concepts.
Exercises: 16, 24, 28, 29, 32

Answers

30. 

31.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 13, 29–34, 42–44

Construct viable arguments and critique the reasoning of others. Exercises 45–46, 51

Model with mathematics. Exercises 35–38

Look for and make use of structure. Exercise 39–41

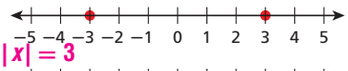
Learn It Online
State Resources Online

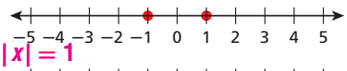
MULTI-STEP TEST PREP **Exercise 44** involves using absolute-value equations to describe water pressures. This exercise prepares students for the Multi-Step Test Prep.

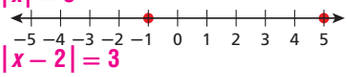
Answers

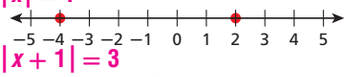
39. When the abs.-value expression is equal to a pos. number, there are 2 solutions. If the abs.-value expression is equal to 0, there is 1 solution, and if the abs.-value expression is equal to a neg. number, there are no solutions.
40. When x is nonneg., the expressions are equal, but when x is neg., the expressions are not equal.
41. Abs. value represents a distance, so it can never be neg.

Write an absolute-value equation whose solutions are graphed on the number line.

35.  $|x| = 3$

36.  $|x| = 1$

37.  $|x - 2| = 3$

38.  $|x + 1| = 3$

Tell whether each statement is sometimes, always, or never true. Explain.

39. An absolute-value equation has two solutions. **sometimes**
40. The value of $|x + 4|$ is equal to the value of $|x| + 4$. **sometimes**
41. The absolute value of a number is nonnegative. **always**
42. **Temperature** A thermostat is set so that the temperature in a laboratory freezer stays within 2.5 °F of 2 °F. Write and solve an absolute-value equation to find the maximum and minimum temperatures in the freezer. $|x - 2| = 2.5$; −0.5 °F; 4.5 °F
43. **Recreation** To ensure safety, boaters must be aware of wind conditions while they are on the water. A particular anemometer gives a measurement of wind speed within a certain amount of the true wind speed, as shown in the table.

Measured Wind Speed (mi/h)	True Wind Speed (mi/h)
20	15–25
22	17–27
24	19–29
26	21–31
28	23–33
30	25–35

43a. $|t - 24| = 5$

b. 19; 29

c. yes, if you change the left side so that the measured wind speed is being subtracted from t

d. The measurements are correct to within 5 mi/h.

a. Use the table to write an absolute-value equation for the minimum and maximum possible true wind speeds t for the measured wind speed shown on the anemometer.

b. Solve your equation from part **a.** Check that the solution is correct by comparing it to the values given in the table when the measured wind speed is 24 mi/h.

c. Will your equation work for all of the values in the table? Explain.

d. Explain what your equation says about the instrument's measurements.

MULTI-STEP TEST PREP

44. The water pumps on a wildland fire apparatus can pump at various rates. The center of the acceptable range of pumping rates is 55 gallons per minute.
    a. Write an absolute-value expression that gives the distance on the number line of a pump's rate r from 55. $|r - 55|$
    b. The smallest and largest pumps have rates that differ by 45 gallons per minute from the rate at the center of the range. Write an absolute-value equation for the rates of these pumps. $|r - 55| = 45$
    c. Find the least and greatest rates that are acceptable for a wildland fire apparatus. **10 gal/min, 100 gal/min**

(cr) David Guttenfelder/AP/Wide World Photos; (b) Index Stock/Alamy

PRACTICE A
PRACTICE C
PRACTICE B

Solve each equation.

1. $|x| = 12$ — {−12, 12}
2. $|x| = \frac{1}{2}$ — $\left\{-\frac{1}{2}, \frac{1}{2}\right\}$
3. $|x| - 6 = 4$ — {−10, 10}
4. $5 + |x| = 14$ — {−9, 9}
5. $3|x| = 24$ — {−8, 8}
6. $|x + 3| = 10$ — {−13, 7}
7. $|x - 1| = 2$ — {−1, 3}
8. $4|x - 5| = 12$ — {−2, 8}
9. $|x + 2| - 3 = 9$ — {−14, 10}
10. $|6x| = 18$ — {−3, 3}
11. $|x - 1| = 0$ — {1}
12. $|x - 3| + 2 = 2$ — {3}
13. How many solutions does the equation $|x + 7| = 1$ have? two
14. How many solutions does the equation $|x + 7| = 0$ have? one
15. How many solutions does the equation $|x + 7| = -1$ have? none

Leticia sets the thermostat in her apartment to 68 degrees. The actual temperature in her apartment can vary from this by as much as 3.5 degrees.

16. Write an absolute-value equation that you can use to find the minimum and maximum temperature. $|x - 68| = 3.5$
17. Solve the equation to find the minimum and maximum temperature. 64.5°; 71.5°

READING STRATEGIES

An absolute-value equation may have two solutions, one solution, or no solution. The following examples show how you can think about an absolute-value equation and make a prediction about the number of solutions.

$|x - 2| = 5$

Think: "What values on a number line are 5 units away from 2?" → There are *two* values that are 5 units away from 2 on the number line.

Prediction: The equation has two solutions.

$7 + |x - 3| = 7$

Think: "7 plus something equals 7, so the absolute-value expression must be equal to zero." → $|x - 3| = 0$ means $x - 3 = 0$. This has only *one* solution, $x = 3$.

Prediction: The equation has one solution.

$|x + 2| = -3$

Think: " What values of x make the absolute-value expression equal to −3?" → An absolute-value expression is *never* negative.

Prediction: The equation has no solutions.

Predict the number of solutions of each absolute-value equation.

1. $6 + |x + 2| = 6$ — one
2. $|x - 10| = 1$ — two
3. $5 + |x - 1| = 0$ — none
4. $2\left|x + \frac{1}{2}\right| = 0$ — one
5. $6|x + 1| = -12$ — none
6. $2 + |x - 4| = 3$ — two

RETEACH

There are three steps in solving an absolute-value equation. First use inverse operations to isolate the absolute-value expression. Then rewrite the equation as two cases that do not involve absolute values. Finally, solve these new equations.

Solve $|x - 3| + 4 = 8$.

Step 1: Isolate the absolute-value expression.

$|x - 3| + 4 = 8$

$-4 \quad -4$ *Subtract 4 from both sides.*

$|x - 3| = 4$

Step 2: Rewrite the equation as two cases.

$|x - 3| = 4$

Case 1	Case 2
$x - 3 = -4$	$x - 3 = 4$
Step 3: Solve. $+3 \quad +3$	$+3 \quad +3$ *Add 3 to both sides.*
$x = -1$	$x = 7$

The solution are −1 and 7.

Solve each equation.

1. $|x - 2| - 3 = 5$ — {−6, 10}
2. $|x + 7| + 2 = 10$ — {−15, 1}
3. $4|x - 5| = 20$ — {0, 10}
4. $|2x| + 1 = 7$ — {−3, 3}

45. Possible answer: no; when an absolute-value expression equals 0, there is only one equation to solve. When an absolute-value expression equals a negative number, there is no equation to solve.

46. No; no matter what value of *a* is chosen, there will always be two solutions: $a + 1$ and $a - 1$.

45. Write About It Do you agree with the following statement: "To solve an absolute-value equation, you need to solve two equations." Why or why not?

46. Critical Thinking Is there a value of a for which the equation $|x - a| = 1$ has exactly one solution? Explain.

TEST PREP

47. Which situation could be modeled by the equation $|x - 65| = 3$?

- (A) Two numbers on the number line are 65 units away from 3.
- (B) The length of a carpet is 3 inches less than 65 inches.
- (C) The maximum and minimum weights of wrestlers on the team are within 3 kg of 65 kg.
- (D) The members of an exercise club for seniors are all between 63 and 67 years old.

48. For which of the following is $n = -3$ a solution?

(F) $|n - 1| = 2$ (G) $|n + 2| = -1$ (H) $|n - 2| = 1$ (J) $|n + 1| = 2$

49. The minimum and maximum sound levels at a rock concert are 90 decibels and 95 decibels. Which equation models this situation?

(A) $|x - 90| = 5$ (B) $|x - 92.5| = 2.5$ (C) $|x - 92.5| = 5$ (D) $|x - 95| = 2.5$

CHALLENGE AND EXTEND

50. 18 in.; the solutions to the equation are 18 and −46, but since *x* represents a length, the negative solution is not reasonable.

50. The perimeter of a rectangle is 100 inches. The length of the rectangle is $|2x - 4|$ inches, and the width is x inches. What are the possible values of x? Explain.

51. Fill in the missing reasons to justify each step in solving the equation $3|2x + 1| = 21$.

Statements	Reasons
1. $3\|2x + 1\| = 21$	**1.** Given
2. $\|2x + 1\| = 7$	**2.** Div. Prop. of Eq.
3. $2x + 1 = -7$ or $2x + 1 = 7$	**3.** Definition of absolute value
4. $2x = -8$ or $2x = 6$	**4.** Subtr. Prop. of Eq.
5. $x = -4$ or $x = 3$	**5.** Div. Prop. of Eq.

52. Solve $|x| = |x + 1|$. (*Hint:* Consider two cases: $x \geq 0$ and $x < 0$.) $-\frac{1}{2}$

COMMON ERROR ALERT

When writing an absolute-value equation of the form $|x - k| = a$, students sometimes confuse the values that should be used for k and a. In **Exercise 42,** students may write $|x - 2.5| = 2$. Explain that the value that represents the variation (in this case, 2.5) should appear alone on one side of the equation, so the correct equation is $|x - 2| = 2.5$.

TEST PREP DOCTOR

In **Exercise 48,** choice **G** can be immediately eliminated because an absolute value is never negative. Therefore, the equation in **G** has no solution.

Journal

Have students write a step-by-step guide to solving absolute-value equations. Ask students to solve a sample equation to illustrate the steps.

ALTERNATIVE ASSESSMENT

Have students write and solve absolute-value equations that have two solutions, one solution, and no solution.

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1-7 Lesson Quiz

Solve each equation.

1. $15 = |x|$ $-15, 15$
2. $2|x - 7| = 14$ $0, 14$
3. $|x + 1| - 9 = -9$ -1
4. $|5 + x| - 3 = -2$ $-6, -4$
5. $7 + |x - 8| = 6$ no solution
6. Inline skates typically have wheels with a diameter of 74 mm. The wheels are manufactured so that the diameters vary from this value by at most 0.1 mm. Write and solve an absolute-value equation to find the minimum and maximum diameters of the wheels. $|x - 74| = 0.1$; 73.9 mm; 74.1 mm

Also available online

PROBLEM SOLVING

Write the correct answer.

1. A machine manufactures wheels with a diameter of 70 cm. It is acceptable for the diameter of a wheel to be within 0.02 cm of this value. Write and solve an absolute-value equation to find the minimum and maximum acceptable diameters.

   $|x - 70| = 0.02$; 69.98 cm; 70.02 cm

2. A pedestrian bridge is 53 meters long. Due to changes in temperature, the bridge may expand or contract by as much as 21 millimeters. Write and solve an absolute-value equation to find the minimum and maximum lengths of the bridge.

   $|x - 53| = 0.021$; 52.979 m; 53.021 m

3. Two numbers on a number line are represented by the absolute-value equation $|n - 5| = 6$. What are the two numbers?

   −1 and 11

4. A jewelry maker cuts pieces of wire to shape into earrings. The equation $|x - 12.2| = 0.3$ gives the minimum and maximum acceptable lengths of the wires in centimeters. What is the minimum acceptable length of a wire?

   11.9 cm

The table shows the recommended daily intake of several minerals for adult women. Use the table for questions 5–7. Select the best answer.

5. Which absolute-value equation gives the minimum and maximum recommended intakes for zinc?

   A $|x - 8| = 32$ C $|x - 16| = 24$
   B $|x - 24| = 16$ D $|x - 40| = 32$

6. For which mineral are the minimum and maximum recommended intakes given by the absolute-value equation $|x - 31.5| = 13.5$?

   F Fluoride H Zinc
   G Iron J None of these

7. Jason writes an equation for the minimum and maximum intakes of fluoride. He writes it in the form $|x - b| = c$. What is the value of b?

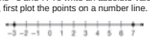
   A 3 C 6.5
   B 3.5 D 7

Mineral	Daily Minimum (mg)	Daily Maximum (mg)
Fluoride	3	10
Iron	18	45
Zinc	8	40

Source: http://www.supplementquality.com/news/multi_mineral_chart.html

CHALLENGE

It is sometimes useful to construct an equation from given solutions. For example, consider the solutions −3 and 7. To write an absolute-value equation with these solutions, first plot the points on a number line.



Find the value that is midway between −3 and 7. One way to do this is to find the mean:

$\frac{-3 + 7}{2} = \frac{4}{2} = 2$

Now find the distance of −3 and 7 from this midpoint. The distance is 5 units.

The required absolute-value equation will be in the form $|x - m| = d$, where m is the midpoint and d is the distance of each solution from the midpoint. Therefore, the absolute-value equation with the solutions −3 and 7 is $|x - 2| = 5$.

1. Show how to check that −3 and 7 are solutions of $|x - 2| = 5$.

   $|(-3) - 2| = |-5| = 5$; $|7 - 2| = |5| = 5$

2. Are there other absolute-value equations with the same solutions? If so, give an example of such an equation. (*Hint:* Consider performing the same operation on each side of the equation $|x - 2| = 5$.)

   yes; possible answer: $|x - 2| + 1 = 6$

Write an absolute-value equation with the given solutions.

3. 1, 9 $|x - 5| = 4$
4. −4, 8 $|x - 2| = 6$
5. −5, 11 $|x - 3| = 8$
6. −9, −1 $|x + 5| = 4$
7. $-\frac{1}{3}, \frac{1}{3}$ $|x| = \frac{1}{3}$
8. 5, 6 $|x - 5.5| = 0.5$
9. −4.5, 9.5 $|x - 2.5| = 7$
10. $-\frac{1}{2}, \frac{5}{2}$ $|x - 1| = \frac{3}{2}$

SECTION 1A

MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills in a real-world format.

Online Edition

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

MATHEMATICAL PRACTICES

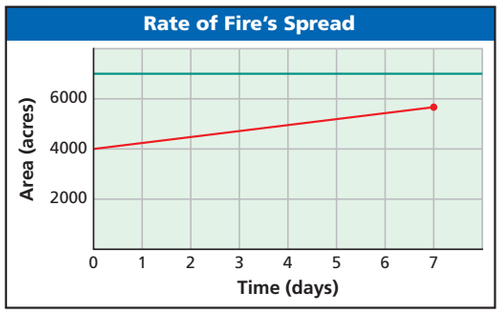
Make sense of problems and persevere in solving them.

Equations and Formulas

All Fired Up A large forest fire in the western United States burns for 14 days, spreading to cover approximately 3850 acres. Firefighters do their best to contain the fire, but hot temperatures and high winds may prompt them to request additional support.

1. The fire spreads at an average rate of how many acres per day? 275 acres
2. Officials estimate that the fire will spread to cover 9075 acres before it is contained. At this rate, how many more days will it take for the fire to cover an area of 9075 acres? Answer this question using at least two different methods. 19 days
3. Additional help arrives, and the firefighters contain the fire in 7 more days. In total, how many acres does the fire cover before it is contained? 5775 acres
4. If the fire had spread to cover an area of 7000 acres, it would have reached Bowman Valley. Explain how the graph shows that firefighters stopped the spread of the fire before it reached Bowman Valley.

The red line does not reach the green horizontal line, which represents 7000.



5. The total cost of fighting the fire for 21 days was approximately $1,440,000. What was the approximate cost per acre of fighting the fire? about $250

(t),©Index Stock/Alamy; (cr),© Francis Specker/Corbis

INTERVENTION

Scaffolding Questions

1. What words indicate a *rate* of spread? acres per day
2. How many acres have already been burned by the fire? 3850 What is the rate of spread of the fire that you found in **Problem 1**? 275 acres per day
3. How many acres were covered during the first 14 days? 3850 How can you find the total number of acres covered during the 21 days? Add 275(7) to the number of acres already burned, 3850.
4. What does the horizontal axis represent? number of days What does the vertical axis represent? number of acres
5. What information do you need to find the cost per acre? total cost and total number of acres burned

Extension

How many acres of land were saved by bringing in additional firefighters? How much time was saved by bringing in additional firefighters? 3300 acres; 12 days

Learn It Online
State Resources Online

READY TO GO ON?

Quiz for Lessons 1-1 Through 1-7

1-1 Variables and Expressions

Give two ways to write each algebraic expression in words.

1. $4 + n$
2. $m - 9$
3. $\frac{g}{2}$
4. $4z$

1-2 Solving Equations by Adding or Subtracting

Solve each equation.

5. $x - 32 = -18$ **14**
6. $1.1 = m - 0.9$ **2**
7. $j + 4 = -17$ **−21**
8. $\frac{9}{8} = g + \frac{1}{2}$ **$\frac{5}{8}$**
9. When she first purchased it, Soledad's computer had 400 GB of hard drive space. After six months, there were only 313 GB available. Write and solve an equation to find the amount of hard drive space that Soledad used in the first six months.
$313 + s = 400$; 87 GB

1-3 Solving Equations by Multiplying or Dividing

Solve each equation.

10. $\frac{h}{3} = -12$ **−36**
11. $-2.8 = \frac{w}{-3}$ **8.4**
12. $42 = 3c$ **14**
13. $-0.1b = 3.7$ **−37**
14. A fund-raiser raised \$2400, which was $\frac{3}{5}$ of the goal. Write and solve an equation to find the amount of the goal. **$\frac{3}{5}g = 2400$; \$4000**

1-4 Solving Two-Step and Multi-Step Equations

Solve each equation.

15. $2r + 20 = 200$ **90**
16. $\frac{3}{5}k + 5 = 7$ **$\frac{10}{3}$**
17. $5n + 6 - 3n = -12$ **−9**
18. $4(x - 7) = 2$ **7.5**
19. A taxicab company charges \$2.10 plus \$0.80 per mile. Carmen paid a fare of \$11.70. Write and solve an equation to find the number of miles she traveled.
$2.10 + 0.80m = 11.70$; 12 mi

1-5 Solving Equations with Variables on Both Sides

Solve each equation.

20. $4x - 3 = 2x + 5$ **4**
21. $3(2x - 5) = 2(3x - 2)$ **no solution**
22. $2(2t - 3) = 6(t + 2)$ **−9**
23. $7(x + 5) = -7(x + 5)$ **−5**

1-6 Solving for a Variable

24. $x = \frac{12 - 3y}{2}$, or $6 - \frac{3}{2}y$

24. Solve $2x + 3y = 12$ for x.
25. Solve $\frac{x}{r} = v$ for x. **$x = rv$**
26. Solve $5j + s = t - 2$ for t. **$t = 5j + s + 2$**
27. Solve $h + p = 3(k - 8)$ for k.
$k = \frac{h + p}{3} + 8$

1-7 Solving Absolute-Value Equations

Solve each equation.

28. $|r| = 7$ **7, −7**
29. $|h + 4| = 11$ **7, −15**
30. $|2x + 4| = 0$ **−2**
31. $16 = 7|p + 3| + 30$ **no solution**

READY TO GO ON? SECTION 1A

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources
Section Quiz

INTERVENTION Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**

***Ready to Go On?* Online**

READY TO GO ON?

Diagnose and Prescribe

NO INTERVENE

YES ENRICH

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 1A: Worksheets	Online	
Lesson 1-1	1-1 Intervention	Activity 1-1	Diagnose and Prescribe Online
Lesson 1-2	1-2 Intervention	Activity 1-2	
Lesson 1-3	1-3 Intervention	Activity 1-3	
Lesson 1-4	1-4 Intervention	Activity 1-4	
Lesson 1-5	1-5 Intervention	Activity 1-5	
Lesson 1-6	1-6 Intervention	Activity 1-6	
Lesson 1-7	1-7 Intervention	Activity 1-7	

***Ready to Go On? Enrichment*, Section 1A**
Worksheets
Online

SECTION 1B

Proportion and Percent

One-Minute Section Planner

Lesson	Lab Resources	Materials
Lesson 1-8 Rates, Ratios, and Proportions • Write and use ratios, rates, and unit rates. • Write and solve proportions. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab	**Optional** rulers (MK), USA or state map
Lesson 1-9 Applications of Proportions • Use proportions to solve problems involving geometric figures. • Use proportions and similar figures to measure objects indirectly. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab	**Optional** colored pencils (MK), graph paper
Lesson 1-10 Precision and Accuracy • Analyze and compare measurements for precision and accuracy. • Choose an appropriate level of accuracy when reporting measurements. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Optional** rulers (MK), balance scale

Note: If NAEP is checked, the content is tested on either the Grade 8 or the Grade 12 NAEP assessment.

MK = *Manipulatives Kit*

Selected Answers in Student Edition

The answers to the following exercises are included in the Selected Answers section at the back of the Student Edition textbook.

Lesson 8: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 43, 45, 47, 49, 51, 53, 55, 59, 61, 63, 65

Lesson 9: 3, 7, 11, 13, 15, 17, 21, 23, 27, 29

Lesson 10: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27a, 27b, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 65, 67, 69, 71, 73, 75, 77, 79, 81

All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Math Background

COMMON CORE PROFESSIONAL DEVELOPMENT

PROPORTIONS

Lessons 1-8 to 1-9

A *proportion* is an equation that states that two ratios are equivalent. The method for solving a proportion, such as $\frac{x}{3} = \frac{5}{8}$, is the same as the method for solving any equation; it requires that the equation be transformed into simpler equivalent equations via a series of mathematically justifiable steps. For example, a possible first step in solving $\frac{x}{3} = \frac{5}{8}$ is to multiply both sides by $3 \cdot 8$, or 24. This step is justified by the Multiplication Property of Equality. The resulting equation, $8x = 15$, can be solved by dividing both sides by 8 (Division Property of Equality).

Performing the above multiplication step on a general proportion leads to the Cross Products Property: If $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, then $ad = bc$. To see why this is true, multiply both sides of the proportion by *bd.*

$\frac{a}{b} = \frac{c}{d}$	
$(bd)\frac{a}{b} = (bd)\frac{c}{d}$	*Multiplication Prop. of Eq.*
$\frac{bda}{b} = \frac{bdc}{d}$	*Multiply.*
$da = bc$	*Simplify.*
$ad = bc$	*Commutative Prop. of Mult.*

Students often use the Cross Products Property without understanding its underlying mathematics; it is simply a "trick." Understanding the mathematical reasons *why* they may use this property is important for students. More generally, students should realize that there are no tricks in mathematics; every step or operation can be justified with a property, definition, etc.

PROPORTIONAL RELATIONSHIPS, UNIT RATES, AND SLOPE

Lesson 1-8

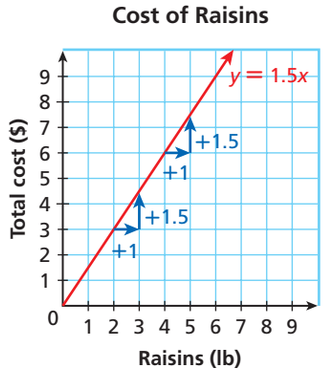
The concept of slope is one of the central ideas in an Algebra 1 course. For students to become fluent in working with slope, they should be able to approach the concept from multiple points of view. Proportions and proportional relationships offer one illuminating entry point.

One way to define a *proportional relationship* is as a relationship that can be expressed as $y = mx$ for a nonzero constant m. A set of ordered pairs (x, y) forms a proportional relationship if $\frac{y}{x}$ is constant for every ordered pair. Students have seen such relationships in a variety of real-world settings. For example, if raisins cost \$1.50 per pound, then $y = 1.5x$ gives the total cost y of x pounds of raisins. The table shows some ordered pairs that satisfy this relationship.

Pounds, *x*	1	2	3	4
Cost (\$), *y*	1.5	3	4.5	6

Notice that $\frac{1.5}{1} = \frac{3}{2} = \frac{4.5}{3} = \frac{6}{4}$. For every ordered pair in the table, $\frac{y}{x} = 1.5$. This is because the *unit rate* for this situation is \$1.50/lb.

The graph of a proportional relationship is a straight line through the origin.



Notice that the slope of the line is 1.5. This means that an increase of one pound causes the total cost to increase by \$1.50.

Students will see several different but equivalent ways of looking at slope: rise over run, change in y divided by change in x, and rate of change. For proportional relationships, slope can also be viewed as a unit rate. In the example, the unit rate is \$1.50/lb, and the slope of the related linear function, $y = 1.5x$, is 1.5. Students will learn that these types of functions are called *direct variations.*

1-8 Organizer

Pacing: Traditional $1\frac{1}{2}$ days
Block $\frac{3}{4}$ day

Objectives: Write and use ratios, rates, and unit rates.

Write and solve proportions.

Algebra Lab
In *Algebra Lab Activities*

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Solve each equation. Check your answer.

1. $6x = 36$ 6

2. $\frac{x}{4} = 12$ 48

3. $5m = 18$ 3.6

4. $\frac{r}{21} = -3$ −63

5. $8y = 18.4$ 2.3

Multiply.

6. $8\left(\frac{7}{8}\right)$ 7 **7.** $12\left(\frac{5}{6}\right)$ 10

Also available online

Math Humor

Q: What did the circle see when sailing on the ocean?

A: Pi rates.

COMMON CORE

CC.9-12.N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas….*

CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems.*

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Learn It Online
State Resources Online

COMMON CORE

1-8 Rates, Ratios, and Proportions

CC.9-12.N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas…* *Also* **CC.9-12.A.CED.1***, **CC.9-12.A.REI.3**

Objectives
Write and use ratios, rates, and unit rates.

Write and solve proportions.

Vocabulary

ratio	proportion
rate	cross products
scale	scale drawing
unit rate	scale model
conversion factor	dimensional analysis

Why learn this?
Ratios and proportions are used to draw accurate maps. (See Example 5.)

A **ratio** is a comparison of two quantities by division. The ratio of a to b can be written $a{:}b$ or $\frac{a}{b}$, where $b \neq 0$. Ratios that name the same comparison are said to be *equivalent*.

A statement that two ratios are equivalent, such as $\frac{1}{12} = \frac{2}{24}$, is called a **proportion**.

EXAMPLE 1 Using Ratios

The ratio of faculty members to students at a college is 1:15. There are 675 students. How many faculty members are there?

$\frac{\text{faculty}}{\text{students}} \rightarrow \frac{1}{15}$ *Write a ratio comparing faculty to students.*

$\frac{1}{15} = \frac{x}{675}$ *Write a proportion. Let x be the number of faculty members.*

$675\left(\frac{x}{675}\right) = 675\left(\frac{1}{15}\right)$ *Since x is divided by 675, multiply both sides of the equation by 675.*

$x = 45$

There are 45 faculty members.

Reading Math
Read the proportion $\frac{1}{15} = \frac{x}{675}$ as "1 is to 15 as x is to 675."

CHECK IT OUT! **1.** The ratio of games won to games lost for a baseball team is 3:2. The team won 18 games. How many games did the team lose? **12**

A **rate** is a ratio of two quantities with different units, such as $\frac{34 \text{ mi}}{2 \text{ gal}}$. Rates are usually written as *unit rates*. A **unit rate** is a rate with a second quantity of 1 unit, such as $\frac{17 \text{ mi}}{1 \text{ gal}}$, or 17 mi/gal. You can convert any rate to a unit rate.

EXAMPLE 2 Finding Unit Rates

Takeru Kobayashi of Japan ate 53.5 hot dogs in 12 minutes to win a contest. Find the unit rate in hot dogs per minute. Round to the nearest hundredth.

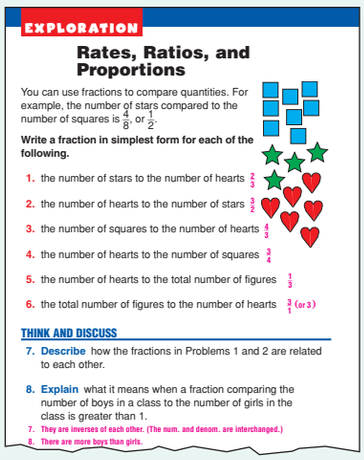
$\frac{53.5}{12} = \frac{x}{1}$ *Write a proportion to find an equivalent ratio with a second quantity of 1.*

$4.46 \approx x$ *Divide on the left side to find x.*

The unit rate is approximately 4.46 hot dogs per minute.

CHECK IT OUT! **2.** Cory earns $52.50 in 7 hours. Find the unit rate in dollars per hour. **$7.50/h**

1 Introduce


EXPLORATION

Rates, Ratios, and Proportions

You can use fractions to compare quantities. For example, the number of stars compared to the number of squares is $\frac{4}{8}$, or $\frac{1}{2}$.

Write a fraction in simplest form for each of the following.

1. the number of stars to the number of hearts $\frac{2}{3}$
2. the number of hearts to the number of stars $\frac{3}{2}$
3. the number of squares to the number of hearts $\frac{4}{3}$
4. the number of hearts to the number of squares $\frac{3}{4}$
5. the number of hearts to the total number of figures $\frac{1}{3}$
6. the total number of figures to the number of hearts $\frac{2}{1}$ (or 3)

THINK AND DISCUSS

7. **Describe** how the fractions in Problems 1 and 2 are related to each other.
8. **Explain** what it means when a fraction comparing the number of boys in a class to the number of girls in the class is greater than 1.

7. They are inverses of each other. (The num. and denom. are interchanged.)
8. There are more boys than girls.

Motivate

Show students the following table of cell phone costs for two different plans.

Plan	Minutes	Cost $
A	800	50
B	600	40

See if students can find the cost per minute for each plan. A: 6.25 cents; B: 6.67 cents Explain that these are *unit rates*, because they are the cost for *one unit* (one minute). Unit rates allow you to compare items of different sizes.

Explorations and answers are provided in your online resources.

Dimensional analysis is a process that uses rates to convert measurements from one unit to another. A rate such as $\frac{12 \text{ in.}}{1 \text{ ft}}$, in which the two quantities are equal but use different units, is called a **conversion factor**. To convert from one set of units to another, multiply by a conversion factor.

EXAMPLE 3 Using Dimensional Analysis

Helpful Hint

In Example 3A Step 1, "pt" appears to divide out, leaving "qt," as the unit. Use this strategy of "dividing out" units when using dimensional analysis.

A **A large adult male human has about 12 pints of blood. Use dimensional analysis to convert this quantity to gallons.**

Step 1 Convert pints to quarts.

$12 \text{ pt} \cdot \frac{1 \text{ qt}}{2 \text{ pt}}$ *Multiply by a conversion factor whose first quantity is quarts and whose second quantity is pints.*

6 qt

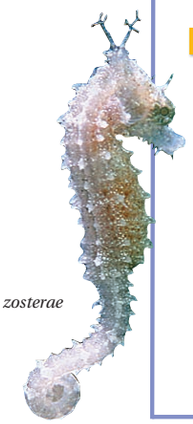
12 pints is 6 quarts.

Step 2 Convert quarts to gallons.

$6 \text{ qt} \cdot \frac{1 \text{ gal}}{4 \text{ qt}}$ *Multiply by a conversion factor whose first quantity is gallons and whose second quantity is quarts.*

$\frac{6}{4} \text{ gal} = 1\frac{1}{2} \text{ gal}$

A large adult male human has about $1\frac{1}{2}$ gallons of blood.



Hippocampus zosterae

B **The dwarf sea horse *Hippocampus zosterae* swims at a rate of 52.68 feet per hour. Use dimensional analysis to convert this speed to inches per minute.**

Use the conversion factor $\frac{12 \text{ in.}}{1 \text{ ft}}$ to convert feet to inches, and use the conversion factor $\frac{1 \text{ h}}{60 \text{ min}}$ to convert hours to minutes.

$$\frac{52.68 \text{ ft}}{1 \text{ h}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} \cdot \frac{1 \text{ h}}{60 \text{ min}} = \frac{10.536 \text{ in.}}{1 \text{ min}}$$

The speed is 10.536 inches per minute.

Check that the answer is reasonable. The answer is about 10 in./min.

- There are 60 min in 1 h, so 10 in./min is $60(10) = 600$ in./h.
- There are 12 in. in 1 ft, so 600 in./h is $\frac{600}{12} = 50$ ft/h. This is close to the rate given in the problem, 52.68 ft/h.

CHECK IT OUT! **3.** A cyclist travels 56 miles in 4 hours. Use dimensional analysis to convert the cyclist's speed to feet per second. Round your answer to the nearest tenth, and show that your answer is reasonable. **20.5 ft/s**

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ and $b \cdot c$ are called **cross products**. You can solve a proportion for a missing value by using the Cross Products Property.

Know it! Note

Cross Products Property

WORDS	NUMBERS	ALGEBRA
In a proportion, cross products are equal.	$\frac{2}{3} \times \frac{4}{6}$ $2 \cdot 6 = 3 \cdot 4$	If $\frac{a}{b} \times \frac{c}{d}$ and $b \neq 0$ and $d \neq 0$, then $ad = bc$.

Shedd Aquarium/Lines Jr./SeaPics.com

COMMON ERROR ALERT

When converting rates, students may write the conversion factor upside down. Stress to students that the unwanted units must "divide out."

Power Presentations with PowerPoint®

Additional Examples

Example 1

The ratio of the number of bones in the human skull to the number of bones in the ears is 11:3. There are 22 bones in the skull. How many bones are in the ears? 6

Example 2

Ralf Laue of Germany flipped a pancake 416 times in 120 seconds to set the world record. Find the unit rate in flips per second. Round to the nearest hundredth. about 3.47 flips/s

Example 3

A. A fast sprinter can run 100 yards in approximately 10 seconds. Use dimensional analysis to convert 100 yards to miles. Round to the nearest hundredth. (*Hint:* There are 1760 yards in a mile.) 0.06 mi

B. A cheetah can run at a rate of 60 miles per hour in short bursts. What is this speed in feet per minute? 5280 ft/min

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- How do you know if the number of students should be written as the first or second quantity in the ratio?
- What does the ratio 1:15 mean?

EXAMPLE 2

- In the ratio $\frac{53.5}{12}$, what does 53.5 represent? What does 12 represent?
- When you solve this proportion, what are you solving for?

EXAMPLE 3

- What is a conversion factor?
- How do you find a conversion factor?

Answer to *Check It Out!*

3. For explanation, see Additional Answers.

2 Teach

Guided Instruction

Before teaching ratios and rates, review how to solve one-step equations with multiplication and division. When solving proportions, remind students to use the Distributive Property when a numerator or denominator has more than one term.

Reaching All Learners *Through Modeling*

Have students work in groups of two or three. Assign each group a ruler (MK) and a task of finding certain measurements (include dimensions for the room, desks, tables, shelves, doors, and so on). Collect and present all measurements. Have students determine a scale that would let them draw a diagram of their classroom on a sheet of paper. Then have them use that scale to draw the diagram.

Power Presentations with PowerPoint®

Additional Examples

Example 4

Solve each proportion.

A. $\frac{3}{9} = \frac{5}{m}$ 15

B. $\frac{6}{y-3} = \frac{2}{7}$ 24

Example 5

A contractor has a blueprint for a house drawn to the scale 1 in:3 ft.

A. A wall on the blueprint is 6.5 inches long. How long is the actual wall? 19.5 ft

B. One wall of the house will be 12 feet long when it is built. How long is the wall on the blueprint? 4 in.

Also available online

Questioning Strategies

EXAMPLE 4

- How could you solve these equations by multiplying both sides by the same number?

EXAMPLE 5

- If the scale is represented by a ratio less than 1, what do we know about the actual object?
- If the scale is represented by a ratio greater than 1, what do we know about the actual object?

Math Background

Students may be curious why the cross-products property works. If you rewrite the ratios with bd as the denominator, then ad and bc will be the equal numerators. For example, $\frac{2}{3} = \frac{4}{6} = \frac{12}{18}$. Here, $\frac{a}{b} = \frac{2}{3}$, $\frac{c}{d} = \frac{4}{6}$, $bd = 18$, and $ad = bc = 12$.

EXAMPLE 4 Solving Proportions

Solve each proportion.

A $\frac{5}{9} = \frac{3}{w}$

$\frac{5}{9} \times \frac{3}{w}$

$5(w) = 9(3)$ *Use cross products.*

$5w = 27$

$\frac{5w}{5} = \frac{27}{5}$ *Divide both sides by 5.*

$w = \frac{27}{5}$

B $\frac{8}{x+10} = \frac{1}{12}$

$\frac{8}{x+10} \times \frac{1}{12}$

$8(12) = 1(x+10)$ *Use cross products.*

$96 = x + 10$

$-10 \quad -10$ *Subtract 10 from both sides.*

$86 = x$

CHECK IT OUT! **Solve each proportion.**

4a. $\frac{-5}{2} = \frac{y}{8}$ **−20**

4b. $\frac{g+3}{5} = \frac{7}{4}$ **5.75**

A **scale** is a ratio between two sets of measurements, such as 1 in:5 mi. A **scale drawing** or **scale model** uses a scale to represent an object as smaller or larger than the actual object. A map is an example of a scale drawing.

EXAMPLE 5 Scale Drawings and Scale Models

A **On the map, the distance from Chicago to Evanston is 0.625 in. What is the actual distance?**

$\frac{\text{map}}{\text{actual}} \rightarrow \frac{1 \text{ in.}}{18 \text{ mi}}$ *Write the scale as a fraction.*

$\frac{1}{18} \times \frac{0.625}{x}$ *Let x be the actual distance.*

$x \cdot 1 = 18(0.625)$ *Use cross products to solve.*

$x = 11.25$

The actual distance is 11.25 mi.

B **The actual distance between North Chicago and Waukegan is 4 mi. What is this distance on the map? Round to the nearest tenth.**

$\frac{\text{map}}{\text{actual}} \rightarrow \frac{1 \text{ in.}}{18 \text{ mi}}$ *Write the scale as a fraction.*

$\frac{1}{18} \times \frac{x}{4}$ *Let x be the distance on the map.*

$4 = 18x$ *Use cross products to solve the proportion.*

$\frac{4}{18} = \frac{18x}{18}$ *Since x is multiplied by 18, divide both sides by 18 to undo the multiplication.*

$0.2 \approx x$ *Round to the nearest tenth.*

The distance on the map is about 0.2 in.

Reading Math

A scale written without units, such as 32:1, means that 32 units of any measure correspond to 1 unit of that same measure.

CHECK IT OUT! **5.** A scale model of a human heart is 16 ft long. The scale is 32:1. How many inches long is the actual heart it represents? **6 in.**

3 Close

Summarize

Review with students how to find unit rates and solve proportions. Caution students to be careful when writing proportions to represent applications. The variable that is the first quantity of one ratio must also be the first quantity of the other. The variable that is the second quantity of one ratio must also be the second quantity of the other.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

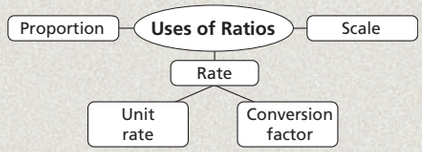
***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

THINK AND DISCUSS

MATHEMATICAL PRACTICES

1. Explain two ways to solve the proportion $\frac{t}{4} = \frac{3}{5}$.
2. How could you show that the answer to Example 5A is reasonable?
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of each use of ratios.



Know it! Note

1-8 Exercises

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Homework Help Online
Parent Resources Online

GUIDED PRACTICE

The ratios are equivalent.

1. **Vocabulary** What does it mean when two ratios form a *proportion*?

SEE EXAMPLE 1

2. The ratio of the sale price of a jacket to the original price is 3:4. The original price is $64. What is the sale price? $48
3. **Chemistry** The ratio of hydrogen atoms to oxygen atoms in water is 2:1. If an amount of water contains 341 trillion atoms of oxygen, how many hydrogen atoms are there? 682 trillion

SEE EXAMPLE 2

4. A computer's fan rotates 2000 times in 40 seconds. Find the unit rate in rotations per second. 50 rotations/s
5. Twelve cows produce 224,988 pounds of milk. Find the unit rate in pounds per cow. 18,749 lb/cow
6. A yellow jacket can fly 4.5 meters in 9 seconds. Find the unit rate in meters per second. 0.5 m/s

SEE EXAMPLE 3

7. Lydia wrote $4\frac{1}{2}$ pages of her science report in one hour. What was her writing rate in pages per minute? 0.075 page/min
8. A model airplane flies 18 feet in 2 seconds. What is the airplane's speed in miles per hour? Round your answer to the nearest hundredth. 6.14 mi/h
9. A vehicle uses 1 tablespoon of gasoline to drive 125 yards. How many miles can the vehicle travel per gallon? Round your answer to the nearest mile. (*Hint:* There are 256 tablespoons in a gallon.) 18 mi/gal

SEE EXAMPLE 4

Solve each proportion.

10. $\frac{3}{z} = \frac{1}{8}$ 24
11. $\frac{x}{3} = \frac{1}{5}$ $\frac{3}{5}$
12. $\frac{b}{4} = \frac{3}{2}$ 6
13. $\frac{f+3}{12} = \frac{7}{2}$ 39
14. $\frac{-1}{5} = \frac{3}{2d}$ −7.5
15. $\frac{3}{14} = \frac{s-2}{21}$ 6.5
16. $\frac{-4}{9} = \frac{7}{x}$ −15.75
17. $\frac{3}{s-2} = \frac{1}{7}$ 23
18. $\frac{10}{h} = \frac{52}{13}$ 2.5

Answers to *Think and Discuss*

1. Possible answer: by using cross-products; by multiplying both sides by 4 to isolate *t*
2. Possible answer: One in. on the map represents 18 mi. So 0.625 in. will represent a little more than half of 18 mi. So 11.25 mi is reasonable.
3. See Additional Answers.

1-8 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–3
Basic 20–25
Average 20–25, 63
Advanced 20–25, 63

If you finished Examples 1–5
Basic 20–41, 44–62
Average 20–38, 40–50, 57–63
Advanced 20–38, 46–65

Homework Quick Check
Quickly check key concepts.
Exercises: 20, 22, 24, 32, 38, 40

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 2–9, 19–25, 38–40, 42–43, 56–58, 63–65

Construct viable arguments and critique the reasoning of others. Exercises 41–42

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Answers

24. Possible answer: 11.9 ft in 3 days is roughly equivalent to 12 ft in 72 h, or $\frac{12}{72} = \frac{1}{6}$ ft/h. There are 12 in. in 1 ft, so this speed is equivalent to $\frac{12}{6} = 2$ in./h. So 1.98 is reasonable.

39. Possible answer: 70 euro will be worth more than $60, so $84 is reasonable.

SEE EXAMPLE 5

19. Archaeology Stonehenge II in Hunt, Texas, is a scale model of the ancient construction in Wiltshire, England. The scale of the model to the original is 3:5. The Altar Stone of the original construction is 4.9 meters tall. Write and solve a proportion to find the height of the model of the Altar Stone.

$\frac{3}{5} = \frac{h}{4.9}$; 2.94 m

Alfred Sheppard, one of the builders of Stonehenge II.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
20–21	1
22–23	2
24–25	3
26–37	4
38	5

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

20. Gardening The ratio of the height of a bonsai ficus tree to the height of a full-size ficus tree is 1:9. The bonsai ficus is 6 inches tall. What is the height of a full-size ficus? 54 in.

21. Manufacturing At one factory, the ratio of defective light bulbs produced to total light bulbs produced is about 3:500. How many light bulbs are expected to be defective when 12,000 are produced? 72

22. Four gallons of gasoline weigh 25 pounds. Find the unit rate in pounds per gallon. 6.25 lb/gal

23. Fifteen ounces of gold cost $6058.50. Find the unit rate in dollars per ounce. $403.90/oz

24. Biology The tropical giant bamboo can grow 11.9 feet in 3 days. What is this rate of growth in inches per hour? Round your answer to the nearest hundredth, and show that your answer is reasonable. 1.98 in./h

25. Transportation The maximum speed of the Tupolev Tu-144 airliner is 694 m/s. What is this speed in kilometers per hour? 2498.4 km/h

Solve each proportion.

26. $\frac{v}{6} = \frac{1}{2}$ 3

27. $\frac{2}{5} = \frac{4}{y}$ 10

28. $\frac{2}{h} = \frac{-5}{6}$ −2.4

29. $\frac{3}{10} = \frac{b+7}{20}$ −1

30. $\frac{5t}{9} = \frac{1}{2}$ 0.9

31. $\frac{2}{3} = \frac{6}{q-4}$ 13

32. $\frac{x}{8} = \frac{7.5}{20}$ 3

33. $\frac{3}{k} = \frac{45}{18}$ 1.2

34. $\frac{6}{a} = \frac{15}{17}$ 6.8

35. $\frac{9}{2} = \frac{5}{x+1}$ $\frac{1}{9}$

36. $\frac{3}{5} = \frac{x}{100}$ 60

37. $\frac{38}{19} = \frac{n-5}{20}$ 45

38. Science The image shows a dust mite as seen under a microscope. The scale of the drawing to the dust mite is 100:1. Use a ruler to measure the length of the dust mite in the image in millimeters. What is the actual length of the dust mite? 0.3 mm

39. Finance On a certain day, the exchange rate was 60 U.S. dollars for 50 euro. How many U.S. dollars were 70 euro worth that day? Show that your answer is reasonable. $84

40. $\frac{20}{85} = \frac{100}{c}$; 425 carp

40. Environmental Science An environmental scientist wants to estimate the number of carp in a pond. He captures 100 carp, tags all of them, and releases them. A week later, he captures 85 carp and records how many have tags. His results are shown in the table. Write and solve a proportion to estimate the number of carp in the pond.

Status	Number Captured
Tagged	20
Not tagged	65

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PRACTICE A
PRACTICE C
PRACTICE B

1. The ratio of freshman to sophomores in a drama club is 5:6. There are 18 sophomores in the drama club. How many freshmen are there? 15

Find each unit rate.

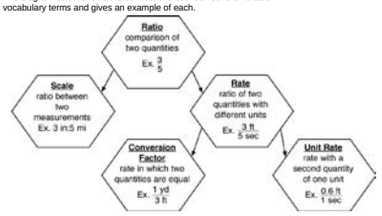
2. Four pounds of apples cost $1.96. $0.49/lb
3. Sal washed 5 cars in 50 minutes. 0.1 cars/min
4. A giraffe can run 32 miles per hour. Use dimensional analysis to convert this speed to feet per second. Round your answer to the nearest tenth. 46.9 ft/s

Solve each proportion.

5. $\frac{y}{4} = \frac{10}{8}$ $y = 5$
6. $\frac{2}{x} = \frac{30}{-6}$ $x = -0.4$
7. $\frac{3}{12} = \frac{-24}{m}$ $m = -96$
8. $\frac{3t}{10} = \frac{1}{2}$ $t = \frac{5}{3}$
9. $\frac{32}{4} = \frac{b+4}{3}$ $b = 20$
10. $\frac{7}{x} = \frac{1}{0.5}$ $x = 3.5$
11. Sam is building a model of an antique car. The scale of his model to the actual car is 1:10. His model is $18\frac{1}{2}$ inches long. How long is the actual car? 185 in.
12. The scale on a map of Virginia shows that 1 centimeter represents 30 miles. The actual distance from Richmond, VA to Washington, DC is 110 miles. On the map, how many centimeters are between the two cities? Round your answer to the nearest tenth. 3.7 cm

READING STRATEGIES

The diagram below shows the connections between several related vocabulary terms and gives an example of each.



Answer each question.

1. There are 16 ounces in a pound. Write this as a conversion factor. $\frac{16 \text{ oz}}{1 \text{ lb}}$ or $\frac{1 \text{ lb}}{16 \text{ oz}}$
2. Is $\frac{1 \text{ ft}}{2 \text{ sec}}$ a unit rate? Explain. no; the second quality is not 1.
3. Why is $\frac{12 \text{ inches}}{1 \text{ foot}}$ a conversion factor? The two quantities are equal.
4. Fill in the blanks with the most appropriate vocabulary terms:
   A rate is a type of ratio.
   A unit rate is a type of rate.
   So, a unit rate is also a type of ratio.
5. Write $\frac{84 \text{ pages}}{3 \text{ hours}}$ as a unit rate. 28 pg/hr
6. A *proportion* is an equation showing two equal ratios. Give an example. Possible answer: $\frac{1}{2} = \frac{12}{24}$

RETEACH

Multiplying by 1 does not change the value of a number: $33 \cdot 1 = 33$.

So, multiplying by a fraction equal to 1 does not change the value either: $33 \cdot \frac{5}{5} = 33$.

Multiplying by 1 is the idea behind converting rates.

Convert 33 feet per second to miles per hour.

Start by converting feet to miles.

$\frac{33 \text{ ft}}{1 \text{ s}}$ *Write the rate as a fraction.*

$\frac{33 \text{ ft}}{1 \text{ s}} \cdot 1 = \frac{33 \text{ ft}}{1 \text{ s}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} = \frac{33 \text{ mi}}{5280 \text{ s}}$

Write the ratio so that feet is in the denominator.

1 mile is equal to 5280 feet, so $\frac{1 \text{ mile}}{5280 \text{ ft}} = 1$.

Now convert seconds to hours.

$\frac{33 \text{ mi}}{5280 \text{ s}} \cdot 1 \cdot 1 = \frac{33 \text{ mi}}{5280 \text{ s}} \cdot \frac{60 \text{ s}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ h}} = \frac{118{,}800 \text{ mi}}{5280 \text{ h}} = 22.5 \text{ mi/h}$

Multiply by 1 twice to convert seconds to minutes and minutes to hours.

Write a fraction equal to 1 for the relationship between these units.

1. feet and yards $\frac{3 \text{ ft}}{1 \text{ yd}} = 1$
2. meters and centimeters $\frac{1}{100} \frac{\text{m}}{\text{cm}} = 1$
3. fluid ounces and cups $\frac{8}{1} \frac{\text{fl oz}}{\text{c}} = 1$
4. The 2004 Tour de France was 3391.1 kilometers. Lance Armstrong won the race in a little over 83.5 hours. Fill in the blanks to find his average speed in meters per hour.

$\frac{3391.1 \text{ km}}{83.5 \text{ h}} \cdot 1 = \frac{3391.1 \text{ km}}{83.5 \text{ h}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = \frac{3{,}391{,}100 \text{ m}}{83.5 \text{ h}} = 40{,}612 \text{ m/h}$

5. A soft-serve ice cream machine makes 1200 gallons per hour. Convert this rate to cups per minute. (*Hint:* 1 gallon is equal to 16 cups.) 320 c/min

41. Possible answer: The denominator in the first ratio is the number of varsity members, but in the second ratio it is the total number of people on the team.

41. ///ERROR ANALYSIS/// Below is a bonus question that appeared on an algebra test and a student's response.

> The ratio of junior varsity members to varsity members on the track team is 3:5. There are 24 members on the team. Write a proportion to find the number of junior varsity members. $\frac{3}{5} = \frac{x}{24}$

The student did not receive the bonus points. Why is this proportion incorrect?

LINK

Sports

The records for the women's 100-meter dash and the women's 200-meter dash were set by Florence Griffith-Joyner, known as "Flo Jo." She is still referred to as the world's fastest woman.

42. Sports The table shows world record times for women's races of different distances.

World Records (Women)	
Distance (m)	Time (s)
100	10.5
200	21.3
800	113.3
5000	864.7

a. Find the speed in meters per second for each race. Round your answers to the nearest hundredth.

b. Which race has the fastest speed? the slowest?

c. Critical Thinking Give a possible reason why the speeds are different.

43. Entertainment Lynn, Faith, and Jeremy are film animators. In one 8-hour day, Lynn rendered 203 frames, Faith rendered 216 frames, and Jeremy rendered 227 frames. How many more frames per hour did Faith render than Lynn did? **1.625**

Solve each proportion.

44. $\frac{x-1}{3} = \frac{x+1}{5}$ **4** **45.** $\frac{m}{3} = \frac{m+4}{7}$ **3** **46.** $\frac{1}{x-3} = \frac{3}{x-5}$ **2** **47.** $\frac{a}{2} = \frac{a-4}{30}$ $-\frac{2}{7}$

48. $\frac{3}{2y} = \frac{16}{y+2}$ $\frac{6}{29}$ **49.** $\frac{n+3}{5} = \frac{n-1}{2}$ $\frac{11}{3}$ **50.** $\frac{1}{y} = \frac{1}{6y-1}$ $\frac{1}{5}$ **51.** $\frac{2}{n} = \frac{4}{n+3}$ **3**

52. $\frac{5t-3}{-2} = \frac{t+3}{2}$ **0** **53.** $\frac{3}{d+3} = \frac{4}{d+12}$ **24** **54.** $\frac{3x+5}{14} = \frac{x}{3}$ **3** **55.** $\frac{5}{2n} = \frac{8}{3n-24}$ -120

56. Decorating A particular shade of paint is made by mixing 5 parts red paint with 7 parts blue paint. To make this shade, Shannon mixed 12 quarts of blue paint with 8 quarts of red paint. Did Shannon mix the correct shade? Explain.

57. Write About It Give three examples of proportions. How do you know they are proportions? Then give three nonexamples of proportions. How do you know they are not proportions?

MULTI-STEP TEST PREP

58. a. Marcus is shopping for a new jacket. He finds one with a price tag of \$120. Above the rack is a sign that says that he can take off $\frac{1}{5}$. Find out how much Marcus can deduct from the price of the jacket. **\$24**

b. What price will Marcus pay for the jacket? **\$96**

c. Copy the model below. Complete it by placing numerical values on top and the corresponding fractional parts below.

d. Explain how this model shows proportional relationships.

(cl) © Duomo/CORBIS; (b) PhotoDisc/gettyimages

MULTI-STEP TEST PREP

Exercise 58 involves using proportions to find sale prices. This exercise prepares students for the Multi-Step Test Prep.

Answers

42a. 9.52 m/s; 9.39 m/s; 7.06 m/s; 5.78 m/s

b. 100 m; 5000 m

c. Possible answer: Runners can maintain a very high speed for a short distance, but over a longer distance their speed drops.

56. No; $\frac{5}{7}$ is not proportional to $\frac{8}{12}$.

57. Possible answer: $\frac{1}{2} = \frac{3}{6}$; $\frac{1}{4} = \frac{5}{20}$; $\frac{3}{7} = \frac{6}{14}$; they are proportions because the cross products are equal; $\frac{1}{2} \neq \frac{5}{6}$; $\frac{1}{2} \neq \frac{6}{7}$; $\frac{3}{4} \neq \frac{2}{3}$; they are not proportions because their cross products are not equal.

58d. Possible answer: It shows that $\frac{1}{5}$ out of 1 is the same as 24 out of 120, $\frac{2}{5}$ out of 1 is the same as 48 out of 120, etc.

PROBLEM SOLVING

Write the correct answer.

1. A donut shop bakes 4 dozen donuts every 18 minutes. Find the unit rate to the nearest hundredth.

2.67 donuts/minute

2. At one time, the ratio of in-state to out-of-state tuition at Texas A & M University in College Station, Texas was about 3:11. About how much was the out-of-state tuition if the in-state tuition at that time was about \$2400?

\$8800

3. The birth rate in Namibia is 35 babies to every 1000 people. In 2001, the country had a population of about 1,800,000 people. How many babies were there?

63,000 babies

4. A boat travels 160 miles in 5 hours. What is its speed in miles per minute?

0.53 mi/min

Use the table below to answer questions 5–8. Select the best answer.
The table shows the ratio of female to male students at various institutions in 2002.

Institution	female:male
Massachusetts Institute of Technology	41:59
Tulane University	53:47
US Naval Academy	1:19
Georgia Institute of Technology	29:71
University of Massachusetts at Amherst	51:49
Baylor University	29:21

5. If there are 209 women at the US Naval Academy, how many men are there?

A 11 C **3971** B 190 D 4180

6. If there are 7282 male students at the Georgia Institute of Technology, how many females are there?

F 2427 H 8282 **G 2974** J 17,828

7. If there are 4959 male students at Baylor University, which proportion can be used to find the number of female students?

A $\frac{21}{4959} = \frac{x}{21}$ C $\frac{21}{29} = \frac{x}{4959}$ B $\frac{21}{4959} = \frac{x}{29}$ **D** $\frac{29}{21} = \frac{x}{4959}$

8. For which institution is the ratio of female to male students the greatest?

F Baylor University
G Tulane University
H University of Massachusetts at Amherst
J US Naval Academy

CHALLENGE

In Lesson 2-7, you learned that in a true proportion, the cross products are equal.

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

The proof to the statement above is shown at right.

$\frac{a}{b} = \frac{c}{d}$

$bd\left(\frac{a}{b}\right) = bd\left(\frac{c}{d}\right)$ Multiplication Property of Equality

$da = bc$ Inverse Property of Multiplication

$ad = bc$ Commutative Property of Multiplication

Using $\frac{a}{b} = \frac{c}{d}$, you can write and prove that other proportions involving a, b, c, and d are true.

In each proportion, assume that the denominator is not equal to 0.

1. Let $\frac{a}{b} = \frac{c}{d}$, where $a = 5$, $b = 7$, $c = 15$, and $d = 21$. Verify that the cross products are equal.

bc: 7 • 15 = 105; *ad*: 5 • 21 = 105

2. a. Let $\frac{a}{b} = \frac{c}{d}$. What can be done to each side of $\frac{a}{b} = \frac{c}{d}$, in order to get $\frac{d}{c} = \frac{b}{a}$?

Multiply each side by $\frac{bd}{ac}$

b. Justify each step in the reasoning below in order to show that $\frac{d}{c} = \frac{b}{a}$.

$\frac{a}{b} = \frac{c}{d}$

$ad = bc$ Cross-Product Property;

$\left(\frac{1}{ac}\right)ad = \left(\frac{1}{ac}\right)bc$ Multiplication Property of Equality;

$\frac{d}{c} = \frac{b}{a}$ Identity Property of Multiplication

3. Let $\frac{a}{b} = \frac{c}{d}$. Show that $\frac{d}{b} = \frac{c}{a}$. Write your reasons to the right of your steps.

$\frac{a}{b} = \frac{c}{d}$

$ad = bc$ Cross-Product Property;

$\left(\frac{1}{ab}\right)ad = \left(\frac{1}{ab}\right)bc$ Multiplication Property of Equality;

$\frac{d}{b} = \frac{c}{a}$ Identity Property of Multiplication

TEST PREP DOCTOR In **Exercise 60,** students who chose **H** or **J** may have merely multiplied or divided 4 and 20. Have students think of Brett's speed as 4 mi per 60 min, or 1 mi per 15 min. Therefore after 20 min (just over 15 min), Brett will be at 1.3 mi, which is just over a mile.

Teaching Tip **Geometry** For **Exercise 63,** point out that if the ratios of the angle measures are 4:5, then one is 4 times a number and the other is 5 times the same number. This leads to the equation $4x + 5x = 90$, which students can solve.

Journal

Have students write about how to find and use unit rates to get the best deal.

ALTERNATIVE ASSESSMENT

Have students use a map to determine the actual distances between three towns. Then have them plan a trip to visit all three towns and return to their starting point. They should also explain how much more gas a vehicle that gets 12 mi/gal will need than a vehicle that gets 25 mi/gal.

Power Presentations with PowerPoint®

1-8 Lesson Quiz

1. In a school, the ratio of boys to girls is 4:3. There are 216 boys. How many girls are there? 162
2. Nuts cost $10.75 for 3 pounds. Find the unit rate in dollars per pound. $3.58/lb
3. Sue washes 25 cars in 5 hours. Find the unit rate in cars per hour. 5 cars/h
4. A car travels 180 miles in 4 hours. Use dimensional analysis to convert the car's speed to feet per minute. 3960 ft/min

Solve each proportion.

5. $\frac{8}{12} = \frac{g}{9}$ 6
6. $\frac{3}{z-4} = \frac{2}{8}$ 16
7. A scale model of a car is 9 inches long. The scale is 1:18. How many inches long is the car it represents? 162 in.

Also available online

TEST PREP

59. One day the U.S. dollar was worth approximately 100 yen. An exchange of 2500 yen was made that day. What was the value of the exchange in dollars?
   (A) $25 (B) $400 (C) $2500 (D) $40,000

60. Brett walks at a speed of 4 miles per hour. He walks for 20 minutes in a straight line at this rate. Approximately what distance does Brett walk?
   (F) 0.06 miles (G) 1.3 miles (H) 5 miles (J) 80 miles

61. A shampoo company conducted a survey and found that 3 out of 8 people use their brand of shampoo. Which proportion could be used to find the expected number of users n in a city of 75,000 people?
   (A) $\frac{3}{8} = \frac{75{,}000}{n}$ (B) $\frac{3}{75{,}000} = \frac{n}{8}$ (C) $\frac{8}{3} = \frac{n}{75{,}000}$ (D) $\frac{3}{8} = \frac{n}{75{,}000}$

62. A statue is 3 feet tall. The display case for a model of the statue can fit a model that is no more than 9 inches tall. Which of the scales below allows for the tallest model of the statue that will fit in the display case?
   (F) 2:1 (G) 1:1 (H) 1:3 (J) 1:4

CHALLENGE AND EXTEND

63. **Geometry** Complementary angles are two angles whose measures add up to 90°. The ratio of the measures of two complementary angles is 4:5. What are the measures of the angles? 40°; 50°

64. A customer wanted 24 feet of rope. The clerk at the hardware store used what she thought was a yardstick to measure the rope, but the yardstick was actually 2 inches too short. How many inches were missing from the customer's piece of rope? 16 in.

65. **Population** The population density of Jackson, Mississippi, is 672.2 people per square kilometer. What is the population density in people per square meter? Show that your answer is reasonable. (*Hint:* There are 1000 meters in 1 kilometer. How many square meters are in 1 square kilometer?) 0.0006722 people/m^2

COMMON CORE

1-9 Applications of Proportions

CC.9-12.N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas.... * *Also* **CC.9-12.A.CED.1***, **CC.9-12.A.REI.3**

Objectives

Use proportions to solve problems involving geometric figures.

Use proportions and similar figures to measure objects indirectly.

Vocabulary

similar
corresponding sides
corresponding angles
indirect measurement
scale factor

Why learn this?

Proportions can be used to find the heights of tall objects, such as totem poles, that would otherwise be difficult to measure. (See Example 2.)

Similar figures have exactly the same shape but not necessarily the same size.

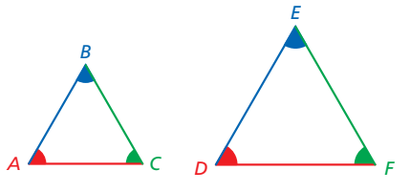
Corresponding sides of two figures are in the same relative position, and **corresponding angles** are in the same relative position. Two figures are similar if and only if the lengths of corresponding sides are proportional and all pairs of corresponding angles have equal measures.

Reading Math

- $\overline{AB}$ means segment AB. AB means the length of $\overline{AB}$.
- $\angle A$ means angle A. $m\angle A$ means the measure of angle A.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$m\angle A = m\angle D$$
$$m\angle B = m\angle E$$
$$m\angle C = m\angle F$$



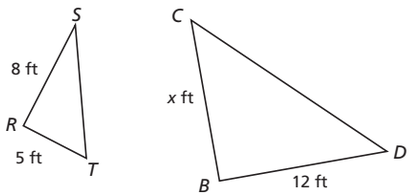
When stating that two figures are similar, use the symbol ~. For the triangles above, you can write $\triangle ABC \sim \triangle DEF$. Make sure corresponding vertices are in the same order. It would be incorrect to write $\triangle ABC \sim \triangle EFD$.

You can use proportions to find missing lengths in similar figures.

EXAMPLE 1 Finding Missing Measures in Similar Figures

Find the value of x in each diagram.

A $\triangle RST \sim \triangle BCD$



R corresponds to B, S corresponds to C, and T corresponds to D.

$$\frac{5}{12} = \frac{8}{x}$$
$$5x = 96$$
$$\frac{5x}{5} = \frac{96}{5}$$
$$x = 19.2$$

$$\frac{RT}{BD} = \frac{RS}{BC}$$

Use cross products.

Since x is multiplied by 5, divide both sides by 5 to undo the multiplication.

The length of $\overline{BC}$ is 19.2 ft.

Robert Harding Picture Library Ltd/Alamy

1 Introduce

EXPLORATION

Applications of Proportions

You will need a sheet of graph paper for this Exploration.

1. Draw each of the following rectangles on graph paper.

	Width	Length
Rectangle A	4	6
Rectangle B	6	9
Rectangle C	8	12
Rectangle D	10	15

2. What do you notice about the shapes of the rectangles? All have the same shape.
3. What is the ratio of the width of rectangle A to the width of rectangle D? $\frac{4}{10}$
4. What is the ratio of the length of rectangle A to the length of rectangle D? $\frac{6}{15}$
5. What do you notice about the two ratios you calculated? They are equal.

THINK AND DISCUSS

6. **Compare** the ratio of the widths of any two of the rectangles to the ratio of their lengths. What can you say about the ratios? They are equal.

Motivate

Draw and label a 4 by 6 rectangle. Ask for the perimeter and area. 20 units; 24 square units Ask students what they think each dimension should be multiplied by if they want the perimeter and area to be twice as big. Change the dimensions as suggested and find the new perimeter and area to see if the students guessed correctly. Tell students that in this lesson, they will learn how changing the dimensions of shapes affects the perimeter, area, and volume.

Explorations and answers are provided in your online resources.

1-9 Organizer

Pacing: Traditional $1\frac{1}{2}$ days
Block $\frac{3}{4}$ day

Objectives: Use proportions to solve problems involving geometric figures.

Use proportions and similar figures to measure objects indirectly.

Algebra Lab
In *Algebra Lab Activities*

Online Edition
Tutorial Videos, Interactivity

Countdown

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Warm Up

Evaluate each expression for $a = 3$, $b = -2$, and $c = 5$.

1. $4a - b$ 14 **2.** $3b^2 - 5$ 7

3. $ab - 2c$ −16

Solve each proportion.

4. $\frac{6}{m} = \frac{2}{3}$ 9 **5.** $\frac{x}{8} = \frac{4}{5}$ 6.4

Also available online

Math Humor

Q: Why did the variable break up with the constant?

A: The constant was incapable of change.

COMMON CORE

CC.9-12.N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas.... *

CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems.*

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

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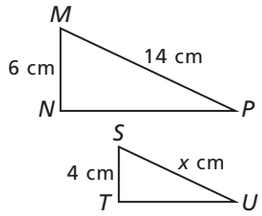
Additional Examples

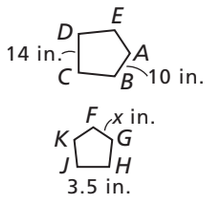
Example 1

Find the value of *x* in each diagram.

A. $\triangle MNP \sim \triangle STU$ $9\frac{1}{3}$



B. $ABCDE \sim FGHJK$ 2.5



Example 2

A flagpole casts a shadow that is 75 ft long at the same time a 6-foot-tall man casts a shadow that is 9 ft long. Write and solve a proportion to find the height of the flag pole. $\frac{75}{9} = \frac{h}{6}$; 50 ft

Also available online

Questioning Strategies

EXAMPLE 1

- If you know the lengths of the sides of two figures, how can you show that the figures are similar?
- What are some ways you can check to see if your answers are reasonable?

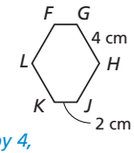
EXAMPLE 2

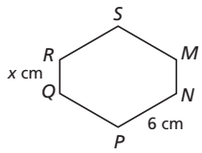
- How do you set up a proportion involving similar triangles?

Teaching Tip **Multiple Representations** There is more than one correct way to set up a proportion. In **Example 2,** students could write $\frac{\text{man's height}}{\text{man's shadow}} = \frac{\text{flagpole's height}}{\text{flagpole's shadow}}$.

Find the value of *x* in each diagram.

B *FGHJKL* ~ *MNPQRS*





$\frac{6}{4} = \frac{x}{2}$ $\frac{NP}{GH} = \frac{RQ}{KJ}$

$4x = 12$ *Use cross products.*

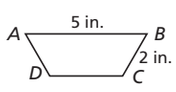
$\frac{4x}{4} = \frac{12}{4}$ *Since x is multiplied by 4, divide both sides by 4 to undo the multiplication.*

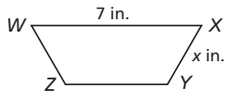
$x = 3$

The length of $\overline{QR}$ is 3 cm.

CHECK IT OUT!

1. Find the value of *x* in the diagram if $ABCD \sim WXYZ$. **2.8 in.**

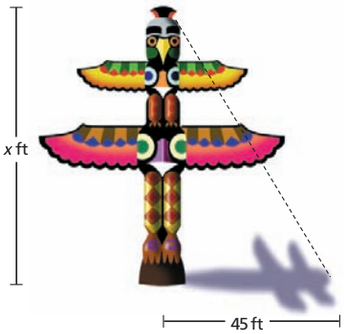




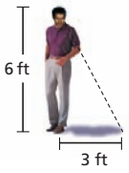
You can solve a proportion involving similar triangles to find a length that is not easily measured. This method of measurement is called **indirect measurement**. If two objects form right angles with the ground, you can apply indirect measurement using their shadows.

EXAMPLE 2 *Measurement Application*

A totem pole casts a shadow 45 feet long at the same time that a 6-foot-tall man casts a shadow that is 3 feet long. Write and solve a proportion to find the height of the totem pole.



Both the man and the totem pole form right angles with the ground, and their shadows are cast at the same angle. You can form two similar right triangles.



Helpful Hint

A height of 90 ft seems reasonable for a totem pole. If you got 900 or 9000 ft, that would not be reasonable, and you should check your work.

$\frac{6}{x} = \frac{3}{45}$

$3x = 270$

$\frac{3x}{3} = \frac{270}{3}$

$x = 90$

$\frac{\text{man's height}}{\text{pole's height}} = \frac{\text{man's shadow}}{\text{pole's shadow}}$

Use cross products. Since x is multiplied by 3, divide both sides by 3 to undo the multiplication.

The totem pole is 90 feet tall.

CHECK IT OUT!

2a. A forest ranger who is 150 cm tall casts a shadow 45 cm long. At the same time, a nearby tree casts a shadow 195 cm long. Write and solve a proportion to find the height of the tree.

2b. A woman who is 5.5 feet tall casts a shadow 3.5 feet long. At the same time, a building casts a shadow 28 feet long. Write and solve a proportion to find the height of the building.

2a. $\frac{150}{x} = \frac{45}{195}$; 650 cm

2b. $\frac{5.5}{x} = \frac{3.5}{28}$; 44 ft

2 Teach

Guided Instruction

Review how to solve proportions before showing how they are applied. In **Example 1,** encourage students to read the statement of similarity before setting up the proportion, because sometimes one of the figures is rotated.

Teaching Tip **Visual** In similarity statements, use colored pencils (MK) to color or circle two corresponding letters to show that they are the endpoints of corresponding sides. In $\triangle ABC \sim \triangle MNO$, $\overline{AC}$ corresponds to $\overline{MO}$.

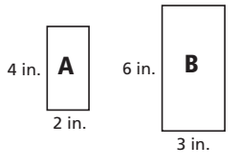
Reaching All Learners *Through World Math*

Have students research the heights of various landmarks around the world. (You may wish to assign a different landmark to each student.) Then have each student write an indirect measurement problem about his or her landmark. Display or distribute these problems and have students solve some of the problems written by other students.

If every dimension of a figure is multiplied by the same number, the result is a similar figure. The multiplier is called a **scale factor**.

EXAMPLE 3 Changing Dimensions

A **Every dimension of a 2-by-4-inch rectangle is multiplied by 1.5 to form a similar rectangle. How is the ratio of the perimeters related to the ratio of corresponding sides? How is the ratio of the areas related to the ratio of corresponding sides?**

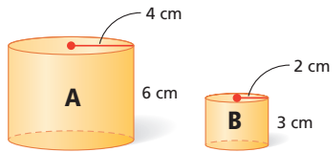


	Rectangle A	Rectangle B
$P = 2\ell + 2w$	$2(2) + 2(4) = 12$	$2(6) + 2(3) = 18$
$A = \ell w$	$4(2) = 8$	$6(3) = 18$

Sides: $\frac{4}{6} = \frac{2}{3}$ Perimeters: $\frac{12}{18} = \frac{2}{3}$ Areas: $\frac{8}{18} = \frac{4}{9} = \left(\frac{2}{3}\right)^2$

The ratio of the perimeters is equal to the ratio of corresponding sides. The ratio of the areas is the square of the ratio of corresponding sides.

B **Every dimension of a cylinder with radius 4 cm and height 6 cm is multiplied by $\frac{1}{2}$ to form a similar cylinder. How is the ratio of the volumes related to the ratio of corresponding dimensions?**



	Cylinder A	Cylinder B
$V = \pi r^2 h$	$\pi(4)^2(6) = 96\pi$	$\pi(2)^2(3) = 12\pi$

Radii: $\frac{4}{2} = \frac{2}{1} = 2$ Heights: $\frac{6}{3} = \frac{2}{1} = 2$ Volumes: $\frac{96\pi}{12\pi} = \frac{8}{1} = 8 = 2^3$

The ratio of the volumes is the cube of the ratio of corresponding dimensions.

Helpful Hint

A scale factor between 0 and 1 reduces a figure. A scale factor greater than 1 enlarges it.

CHECK IT OUT!

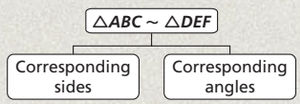
3. A rectangle has width 12 inches and length 3 inches. Every dimension of the rectangle is multiplied by $\frac{1}{3}$ to form a similar rectangle. How is the ratio of the perimeters related to the ratio of the corresponding sides? **The ratio of the perimeters is equal to the ratio of the corresponding sides.**

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Name some pairs of real-world items that appear to be similar figures.

2. GET ORGANIZED Copy and complete the graphic organizer. In the top box, sketch and label two similar triangles. Then list the corresponding sides and angles in the bottom boxes.



Know it! Note

COMMON ERROR ALERT

Students often divide in the wrong order to get the scale factor, and may say that a scale factor is $\frac{1}{4}$ when it is 4. Remind students that a scale factor of greater than 1 enlarges the figure.

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Additional Examples

Example 3

A. The radius of a circle with radius 8 in. is multiplied by 1.75 to get a circle with radius 14 in. How is the ratio of the circumferences related to the ratio of the radii? How is the ratio of the areas related to the ratio of the radii? The ratio of the circumferences is equal to the ratio of the radii. The ratio of the areas is the square of the ratio of the radii.

B. Every dimension of a rectangular prism with length 12 cm, width 3 cm, and height 9 cm is multiplied by $\frac{1}{3}$ to get a similar rectangular prism. How is the ratio of the volumes related to the ratio of the corresponding dimensions? The ratio of the volumes is the cube of the ratio of the corresponding dimensions.

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 3

- What is meant by the dimensions of a rectangular prism?
- If you know the scale factor and perimeter of the original figure, how can you find the perimeter of the new figure? the area of the new figure?

3 Close

Summarize

Have students describe what happens to a scale drawing when the scale of 1 in:50 ft is changed to 1 in:20 ft. The drawing becomes 2.5 times as large.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. Possible answers: a model airplane and the real airplane it represents; a baseball and a softball; a photograph and its enlargement
2. See Additional Answers.

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–3**

Basic 6–17, 20, 22, 24–26
Average 6–13, 14–18 even, 20–27
Advanced 6–12, 14–29

Homework Quick Check
Quickly check key concepts.
Exercises: 6, 8, 9, 10, 12, 14, 21

Answers

10. Possible answer: the width is less than the length for the baby blanket, so the width on the mother's blanket will also be less than the length. Since 5 ft is less than $6\frac{1}{4}$ ft, 5 ft is a reasonable answer.

1-9 Exercises

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Parent Resources Online

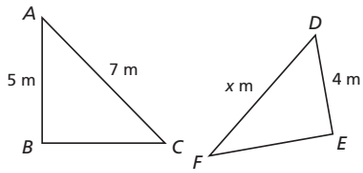
GUIDED PRACTICE

1. **Vocabulary** What does it mean for two figures to be *similar*?
They are the same shape but not necessarily the same size.

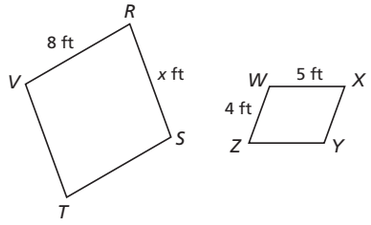
SEE EXAMPLE 1

Find the value of x in each diagram.

2. $\triangle ABC \sim \triangle DEF$ 5.6 m



3. $RSTV \sim WXYZ$ 10 ft



SEE EXAMPLE 2

4. Roger is 5 feet tall and casts a shadow 3.5 feet long. At the same time, the flagpole outside his school casts a shadow 14 feet long. Write and solve a proportion to find the height of the flagpole. $\frac{5}{h} = \frac{3.5}{14}$; 20 ft

SEE EXAMPLE 3

5. A rectangle has length 12 feet and width 8 feet. Every dimension of the rectangle is multiplied by $\frac{3}{4}$ to form a similar rectangle. How is the ratio of the areas related to the ratio of corresponding sides?
The ratio of the areas is the square of the ratio of the corresponding side lengths.

PRACTICE AND PROBLEM SOLVING

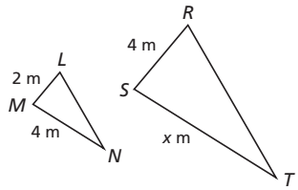
Independent Practice

For Exercises	See Example
6–7	1
8	2
9	3

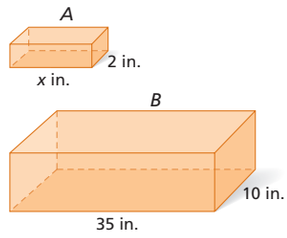
Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Find the value of x in each diagram.

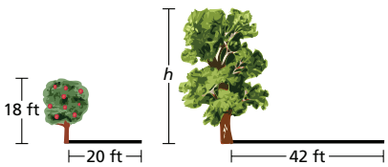
6. $\triangle LMN \sim \triangle RST$ 8 m



7. prism $A \sim$ prism B 7 in.



8. Write and solve a proportion to find the height of the taller tree in the diagram at right. $\frac{18}{h} = \frac{20}{42}$; 37.8 ft



9. A triangle has side lengths of 5 inches, 12 inches, and 15 inches. Every dimension is multiplied by $\frac{1}{5}$ to form a new triangle. How is the ratio of the perimeters related to the ratio of corresponding sides?
The ratio of the perimeters is equal to the ratio of the corresponding side lengths.

10. **Hobbies** For a baby shower gift, Heather crocheted a baby blanket whose length was $2\frac{1}{2}$ feet and whose width was 2 feet. She plans to crochet a proportionally larger similar blanket for the baby's mother. If she wants the length of the mother's blanket to be $6\frac{1}{4}$ feet, what should the width be? Show that your answer is reasonable. 5 ft

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 4–5, 8–13, 18–20, 21–22, 27–29

Reason abstractly and quantitatively. Exercise 23

Construct viable arguments and critique the reasoning of others. Exercise 19

Model with mathematics. Exercises 10–11, 22

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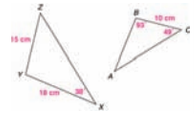
READING STRATEGIES

Use the table below to help you understand the symbols commonly used in geometry.

Symbols	Words
$\overline{XY}$	segment XY
$\angle X$	angle X
$\triangle XYZ$	triangle XYZ
XY	the length of segment XY
$m\angle X$	the measure of angle X
~	is similar to



Answer each question.

1. Write "triangle ABC" in symbols. $\triangle ABC$
2. Write $m\angle B = 93°$ in words. The measure of angle B is 93 degrees.
3. Write $\triangle XYZ \sim \triangle ABC$ in words.
Triangle XYZ is similar to triangle ABC.
4. Use the information given in symbols below to label the diagram shown above.
$m\angle X = 38°$ $m\angle B = 93°$ $m\angle C = 49°$
$XY = 18$ cm $YZ = 15$ cm $BC = 10$ cm

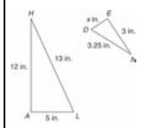
Corresponding angles of similar triangles have the same measure.

5. What is $m\angle Z$? 49° 6. What is $m\angle A$? 38°
7. Write a proportion relating AB, BC, XY, and YZ. Possible answer: $\frac{AB}{BC} = \frac{XY}{YZ}$
8. Substitute the known values into the proportion from problem 7, and then solve it to find AB.
$\frac{X}{10} = \frac{18}{15}$; 12 cm

RETEACH

To find missing lengths in similar polygons, you must know which sides correspond (match). Making a table can help.

$\triangle HAL \sim \triangle NED$. Find the value of x.

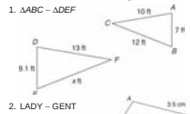


Triangle	First two points	Last two points	First and last points
HAL	*HA*	*AL*	*HL*
	12 in.	5 in.	13 in.
NED	*NE*	*ED*	*ND*
	3 in.	*x* in.	3.25 in.

$\frac{HA}{NE} = \frac{AL}{ED}$ *Choose any two columns from the table to set up a proportion.*
$\frac{12}{3} = \frac{5}{x}$ *Substitute.*
$12x = 15$ *Use cross products.*
$x = 1.25$ *Divide both sides by 12.*

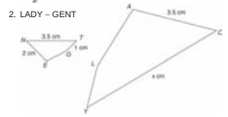
Find the value of x in each diagram.

1. $\triangle ABC \sim \triangle DEF$



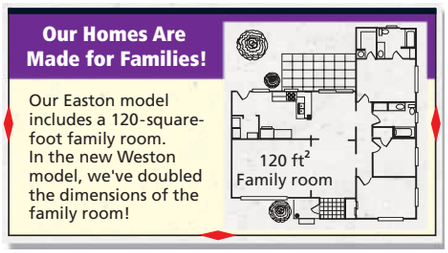
$x = 15.6$ ft

2. LADY ~ GENT



$x = 6.125$ cm

11. Real Estate Refer to the home builder's advertisement. The family rooms in both models are rectangular. How much carpeting is needed to carpet the family room in the Weston model? **480 ft²**

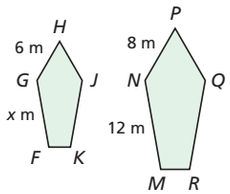


12. A rectangle has an area of 16 ft². Every dimension is multiplied by a scale factor, and the new rectangle has an area of 64 ft². What was the scale factor? **2**

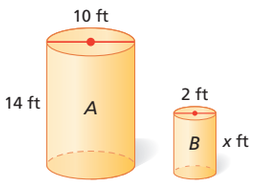
13. A cone has a volume of 98π cm³. Every dimension is multiplied by a scale factor, and the new cone has a volume of 6272π cm³. What was the scale factor? **4**

Find the value of *x* in each diagram.

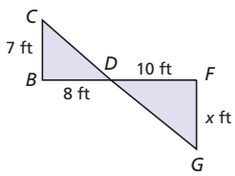
14. $FGHJK \sim MNPQR$ **9 m**



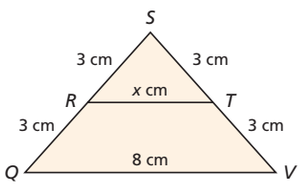
15. cylinder $A \sim$ cylinder B **2.8 ft**



16. $\triangle BCD \sim \triangle FGD$ **8.75 ft**



17. $\triangle RST \sim \triangle QSV$ **4 cm**



18. $\frac{4}{x} = \frac{6}{450}$; **300 ft**

19. Possible answer: No; a 16-in. pizza actually has 4 times the area, so the cost should be 4 times as much.

18. A tower casts a 450 ft shadow at the same time that a 4 ft child casts a 6 ft shadow. Write and solve a proportion to find the height of the tower.

19. Write About It At Pizza Palace, a pizza with a diameter of 8 inches costs $6.00. The restaurant manager says that a 16-inch pizza should be priced at $12.00 because it is twice as large. Do you agree? Explain why or why not.

MULTI-STEP TEST PREP

PhotoDisc/getty images

20. Another common application of proportion is *percents*. A percent is a ratio of a number to 100. For example, $80\% = \frac{80}{100}$.

a. Write 12%, 18%, 25%, 67%, and 98% as ratios. $\frac{12}{100}; \frac{18}{100}; \frac{25}{100}; \frac{67}{100}; \frac{98}{100}$

b. Percents can also be written as decimals. Write each of your ratios from part **a** as a decimal. **0.12; 0.18; 0.25; 0.67; 0.98**

c. What do you notice about a percent and its decimal equivalent?

MULTI-STEP TEST PREP **Exercise 20** involves using ratios to find percents. This exercise prepares students for the Multi-Step Test Prep.

Answers

20c. The decimal is the percent with its decimal point moved 2 places to the left and the % symbol removed.a

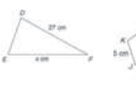
PRACTICE A
PRACTICE C
PRACTICE B

Find the value of *x* in each diagram.

1. $\triangle ABC \sim \triangle DEF$

15

2. $FGHJK \sim MNPQR$



3.2

3. A utility worker is 5.5 feet tall and is casting a shadow 4 feet long. At the same time, a nearby utility pole casts a shadow 20 feet long. Write and solve a proportion to find the height of the utility pole. $\frac{5.5}{x} = \frac{4}{20}$; **27.5 feet**

4. A cylinder has a radius of 3 cm and a length of 10 cm. Every dimension of the cylinder is multiplied by 3 to form a new cylinder. How is the ratio of the volumes related to the ratio of corresponding dimensions?

The ratio of the volumes is the cube of the ratio of the corresponding sides.

5. A rectangle has an area of 48 in². Every dimension of the rectangle is multiplied by a scale factor, and the new rectangle has an area of 12 in².

What was the scale factor? $\frac{1}{2}$

PROBLEM SOLVING

Write the correct answer.

1. A 4 by 5 inch photo is enlarged by multiplying every dimension by 2 to form a similar 8 by 10 inch photo. What is the ratio of the perimeter of the smaller rectangle to that of the larger? What is the ratio of the two areas?

1:2
1:4

2. Pamela wants to buy a suitcase whose dimensions are $1\frac{1}{2}$ times those of her $28 \times 16 \times 8$ inch suitcase. How is the ratio of the volumes related to the ratio of corresponding dimensions? What is the ratio of the volumes?

The ratio of the volumes is the cube of the ratio of the corresponding dimensions; 8:27

3. The Taylors plan to expand their 80 square foot garage by tripling the dimensions. What will be the area of the new garage?

720 sq. ft.

4. A tent has a volume of 26.25 in³. Every dimension is multiplied by a scale factor so that the new tent has a volume of 1680 in³. What was the scale factor?

4

Complete the table below and use it to answer questions 5–8. Select the best answer. Assume the shadow lengths were measured at the same time of day.

Object	Length of Shadow (ft)	Height (ft)
Flagpole	8	20
Oak tree	12	
Goal post	18	
Telephone pole		17.5
Fence		6.5

5. The flagpole casts an 8 foot shadow, as shown in the table. At the same time, the oak tree casts a 12 foot shadow. How tall is the oak tree?

A 4.8 ft C 30 ft
B 24 ft D 32 ft

6. How tall is the goal post?

F 7.2 ft H 38 ft
G 30 ft **J** 45 ft

7. What is the length of the telephone pole's shadow?

A 5.5 ft C 25.5 ft
B 7 ft D 43.8 ft

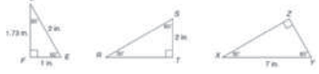
8. What is the length of the fence's shadow?

F 1.5 ft H 16.25 ft
G 2.6 ft J 21.5 ft

CHALLENGE

Trigonometry is the study of angle and side relationships in triangles. Right triangles are especially important in trigonometry. In a right triangle, the side opposite the right angle is always called the **hypotenuse**, and the remaining sides are called the **legs**. Each leg can be described as **opposite** or **adjacent** to a given acute angle.

Use these similar right triangles to fill in the table.



	$\triangle DEF$	$\triangle RST$	$\triangle XYZ$
1. Name the side opposite the 30° angle in each triangle.	$\overline{EF}$	$\overline{ST}$	$\overline{YZ}$
2. What is the length of the side opposite the 30° angle?	1 in.	2 in.	3.5 in.
3. Name the hypotenuse.	$\overline{DE}$	$\overline{RS}$	$\overline{XY}$
4. What is the length of the hypotenuse?	2 in.	4 in.	7 in.
5. Calculate $\frac{\text{length of side opposite } 30°}{\text{length of hypotenuse}}$.	$\frac{1}{2} = 0.5$	$\frac{2}{4} = \frac{1}{2} = 0.5$	$\frac{3.5}{7} = \frac{1}{2} = 0.5$

6. The ratio in problem 5 is called the **sine** of the angle. Based on your results, what is the sine of a 30° angle in any right triangle? $\frac{1}{2}$ or 0.5

7. Use the triangles above to determine the sine of a 60° angle. 0.865

8. The ratio $\frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$ is called the **cosine** of the angle.

a. Use the triangles above to determine the cosine of a 30° angle. 0.865

b. Use the triangles above to determine the cosine of a 60° angle. $\frac{1}{2}$ or 0.5

9. Explain how the sines of 30° and 60° are related to the cosines of 30° and 60°.

The sine of a 30° angle equals the cosine of a 60° angle.
The sine of a 60° angle equals the cosine of 30° angle.

TEST PREP DOCTOR In **Exercise 24,** the radius of the first ball is twice the radius of the second. Since $2^3 = 8$, to find the volume of the smaller sphere, divide 800 by 8.

In **Exercise 25,** students can look at the first ratio in each similarity statement and check that the letters *S* and *G* are in the same positions as *M* and *W*. Only choice **G** has them placed incorrectly.

Journal

Give students the following problem: You and a friend are discussing the height of a tree. Explain how to use proportions to find this height.

ALTERNATIVE ASSESSMENT

Charlene is making a quilt out of rectangular blocks. The pattern for each block is 6 in. by 9 in. If she reduces the blocks to $\frac{2}{3}$ this size and uses 21 blocks for the width and 24 blocks for the length, what will be the dimensions and area of the quilt? Show all of your work and explain how you got your answer.

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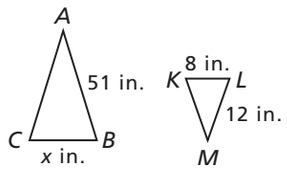
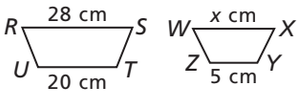
1-9 Lesson Quiz

Find the value of *x* in each diagram.

1. $\triangle ABC \sim \triangle MLK$ 34



2. *RSTU* ~ *WXYZ* 7



3. A girl that is 5 ft tall casts a shadow 4 ft long. At the same time, a tree casts a shadow 24 ft long. How tall is the tree? 30 ft

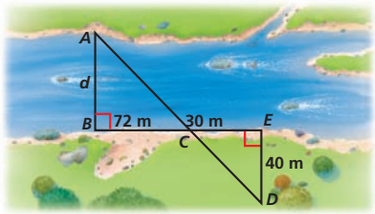
4. The lengths of the sides of a square are multiplied by 2.5. How is the ratio of the areas related to the ratio of the sides? The ratio of the areas is the square of the ratio of the sides.

Also available online

21. $\frac{1.5}{x} = \frac{4.5}{36}$; 12 m

21. A lighthouse casts a shadow that is 36 meters long. At the same time, a person who is 1.5 meters tall casts a shadow that is 4.5 meters long. Write and solve a proportion to find the height of the lighthouse.

22. In the diagram, $\triangle ABC \sim \triangle DEC$. What is the distance across the river from *A* to *B*? 96 m



23. Critical Thinking If every dimension of a two-dimensional figure is multiplied by *k*, by what quantity is the area multiplied? k^2

TEST PREP

24. A beach ball holds 800 cubic inches of air. Another beach ball has a radius that is half that of the larger ball. How much air does the smaller ball hold?

Ⓐ 400 cubic inches
Ⓑ 200 cubic inches
Ⓒ 100 cubic inches
Ⓓ 80 cubic inches

25. For two similar triangles, $\frac{SG}{MW} = \frac{GT}{WR} = \frac{TS}{RM}$. Which statement below is NOT correct?

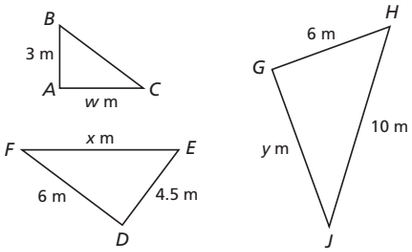
Ⓕ $\triangle SGT \sim \triangle MWR$
Ⓖ $\triangle GST \sim \triangle MRW$
Ⓗ $\triangle TGS \sim \triangle RWM$
Ⓙ $\triangle GTS \sim \triangle WRM$

26. Gridded Response A rectangle has length 5 centimeters and width 3 centimeters. A similar rectangle has length 7.25 centimeters. What is the width in centimeters of this rectangle? 4.35

CHALLENGE AND EXTEND

27. $w = 4$; $x = 7.5$; $y = 8$

27. Find the values of *w*, *x*, and *y* given that $\triangle ABC \sim \triangle DEF \sim \triangle GHJ$.



28. $\triangle RST \sim \triangle VWX$ and $\frac{RT}{VX} = b$.

What is $\frac{\text{area of } \triangle RST}{\text{area of } \triangle VWX}$? b^2

29. Multi-Step Rectangles *A* and *B* are similar. The area of *A* is 30.195 cm². The length of *B* is 6.1 cm. Each dimension of *B* is $\frac{2}{3}$ the corresponding dimension of *A*. What is the perimeter of *B*? 16.6 cm

COMMON CORE

1-10 Precision and Accuracy

CC.9-12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*
Also **CC.9-12.N.Q.2***

Objectives
Analyze and compare measurements for precision and accuracy.

Choose an appropriate level of accuracy when reporting measurements.

Vocabulary
precision
accuracy
tolerance

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Who uses this?

Chemists must understand precision and accuracy when weighing or mixing specific amounts of chemicals. (See Example 2.)

When you measure an object, you must use an instrument that will give an appropriate measurement. A scale to measure the mass of a person may show mass to the nearest kilogram. A scale to measure chemicals in a lab may show mass to the nearest milligram.

Precision is the level of detail in a measurement and is determined by the smallest unit or fraction of a unit that you can reasonably measure. Sometimes, the instrument determines the precision of a measurement. At other times, measurements are rounded to a specified precision.

A scale that shows the mass of an object to the nearest milligram is more precise than a scale that shows the mass of an object to the nearest kilogram, because a milligram is a smaller unit of measure than a kilogram. Likewise, a scale that shows the mass of an object as 24.23 grams is more precise than a scale that shows the mass of the same object as 24.2 grams.

EXAMPLE 1 Comparing Precision of Measurements

Choose the more precise measurement in each pair.

A **3.4 kg; 3421 g**

3.4 kg	*Nearest tenth of a kilogram*
3421 g	*Nearest gram*

A gram is smaller than a tenth of a kilogram, so 3421 g is more precise.

B **3.4 cm; 3.43 cm**

3.4 cm	*Nearest tenth of a centimeter*
3.43 cm	*Nearest hundredth of a centimeter*

A hundredth of a centimeter is smaller than a tenth of a centimeter, so 3.43 cm is more precise.

C **3 ft; 36 in.**

3 ft	*Nearest foot*
36 in.	*Nearest inch*

An inch is smaller than a foot, so 36 in. is more precise.

1-10 Organizer

Pacing: Traditional 2 days
Block 1 day

Objectives: Analyze and compare measurements for precision and accuracy.

Choose an appropriate level of accuracy when reporting measurements.

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Warm Up

Convert each measure.

1. 3210 mm to centimeters 321 cm
2. 18 in. to feet 1.5 ft
3. 52.5 kg to grams 52,500 g
4. 2.5 lbs to ounces 40 oz

Find each absolute value.

5. $|-2|$ 2
6. $|8.1|$ 8.1
7. $|3 - 1.2|$ 1.8
8. $|7 - 10|$ 3

Also available online

Math Humor

Soccer coach: You're a terrible player. You scored five goals for the other team! What do you have to say for yourself?

Soccer player: Well, my accuracy needs work, but my precision is great!

COMMON CORE

CC.9-12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*
CC.9-12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.*

Learn It Online
State Resources Online

1 Introduce

EXPLORATION

Precision and Accuracy

1. In darts, an accurate throw hits the center of the board, called the bull's eye. Are the throws shown accurate? Why or why not? No; the throws are not in the bull's eye.
2. The distance of each dart to the center of the bull's eye is measured as 3.4 in., 3.52 in., and 3.518 in. The most precise measurement has the greatest level of detail. Which measurement is the most precise? Why? 3.518 in.; this measurement is to a thousandth of an inch, the smallest or most detailed value shown.
3. The diameter of a regulation dart board is 17.75 in. Three people measure a regulation dart board and get the following results: 18 in., 17.875 in., and 17.75 in. Which measurement is most accurate? Which is most precise? Most accurate: 17.75 in.; most precise: 17.875 in.

THINK AND DISCUSS

4. **Explain** how to find the more precise measurement when the units are different. Possible answer: Convert the measurements to the same units and determine which has the greater level of detail. If the level of detail is the same, the original measurement with the smaller unit is more precise.
5. **Describe** how a measurement with a particular ruler could be more precise but not as accurate as a measurement with a different ruler. Possible answer: One ruler could be more precise than another if it measures to a smaller unit, but it could be less accurate if it is read incorrectly.

Motivate

Draw two diagrams as shown. Explain that they are the results of two players' attempts to hit a bull's eye in a game of darts. Have students compare and contrast the throwing skills of each player. Explain that they will learn how to articulate the differences in the players' skills in this lesson.

Player A

Player B

Explorations and answers are provided in your online resources.

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Additional Examples

Example 1

Choose the more precise measurement in each pair.

A. 0.8 km; 830.2 m 830.2 m

B. 2.45 in.; 2.5 in. 2.45 in.

C. 100 cm; 1 m 100 cm

Example 2

Ida works in a deli. She is testing the scales at the deli to make sure they are accurate. She uses a weight that is *exactly* 1 pound and gets the following results:

Scale 1: 1.019 lb

Scale 2: 1.01 lb

Scale 3: 0.98 lb

a. Which scale is the most precise? 1

b. Which scale is the most accurate? 2

Also available online

Questioning Strategies

EXAMPLE 1

- Will you always get a more precise measurement by using smaller units, say centimeters rather than meters?

EXAMPLE 2

- If a package of deli meat measures x on scale 3, what predictions can you make about the measures of its weight on scales 1 and 2?

Teaching Tip

Critical Thinking

Techniques used to ensure accuracy in experiments include checking a known mass on a scale, measuring twice, and using two different tools or methods. Ask students to think of other possible techniques.

CHECK IT OUT! **Choose the more precise measurement in each pair.**

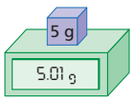
1a. 2 lb; 17 oz **1b.** 7.85 m; 7.8 m **1c.** 6 kg; 6000 g

A precise measurement is only useful if the measurement is also *accurate*. The **accuracy** of a measurement is the closeness of a measured value to the actual or true value. Two measurement tools may measure to the same precision, but not have the same accuracy. Similarly, using a more precise measuring instrument will not necessarily give a more accurate measurement.

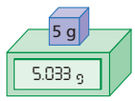
EXAMPLE 2 Comparing Precision and Accuracy

Sam is a technician in a pharmaceutical lab. Each week, she must test the scales in the lab to make sure they are accurate. She uses a standard mass that is *exactly* 5.000 grams and gets the following results:

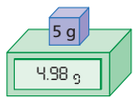
Scale 1



Scale 2



Scale 3



a. Which scale is the most precise?

Scales 1 and 3 measure to the nearest hundredth of a gram.

Scale 2 measures to the nearest thousandth of a gram.

Because a thousandth of a gram is smaller than a hundredth of a gram, Scale 2 is the most precise.

b. Which scale is the most accurate?

For each scale, find the absolute value of the difference of the standard mass and the scale reading.

Scale 1: $|5.000 - 5.01| = 0.01$

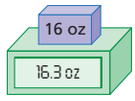
Scale 2: $|5.000 - 5.033| = 0.033$

Scale 3: $|5.000 - 4.98| = 0.02$

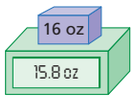
Because $0.01 < 0.02 < 0.033$, Scale 1 is the most accurate.

CHECK IT OUT! **2.** A standard mass of 16 ounces is used to test three postal scales. The results are shown below.

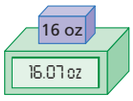
Scale A



Scale B



Scale C



a. Which scale is the most precise? C

b. Which scale is the most accurate? C

When you measure a group of objects that are expected to be similar, you may find that there are variations from the expected value. **Tolerance** describes the amount by which a measurement is permitted to vary from a specified value. Tolerance is often expressed as a range of values, such as 5 mm ± 0.3 mm, which is equivalent to 4.7 mm–5.3 mm.

2 Teach

Guided Instruction

This lesson will help students understand the significance of measurements while doing mathematical data analysis. Review how to convert between commonly used units of mass, length, and weight, as well as the concept of absolute value. Draw students' attention to the number of decimal places to which the measurements are made. Be sure that students grasp the difference between precision and accuracy.

Reaching All Learners

Through Kinesthetic Experience

Pair students and give each pair a meter stick. Have both students in each pair measure the same object. Each student may pick any of the following units to measure to: millimeter, centimeter, tenth of a centimeter, meter, tenth of a meter, or hundredth of a meter. Have each pair of students compare their results and say whether one measurement is more precise.

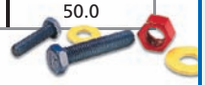
EXAMPLE 3 Using a Specified Tolerance

Writing Math

The final zero in a decimal measurement such as 50.0 mm should not be dropped. 50.0 mm indicates a precision of one-tenth of a millimeter. 50 mm indicates a precision of one millimeter, and is less precise than 50.0 mm.

Acme Nuts & Bolts is manufacturing a bolt to use in an airplane. The length of the bolt should be 50 mm, with a tolerance of 0.5 mm (50 mm ± 0.5 mm). A batch of bolts had the lengths shown in the table. Do all of the bolts measure within the specified tolerance? If not, which bolt(s) are not within the specified tolerance?

Bolt	Length (mm)
A	49.8
B	50.4
C	49.5
D	50.1
E	49.4
F	50.0



$50 - 0.5 = 49.5$ *50 mm ± 0.5 mm means that the*

$50 + 0.5 = 50.5$ *bolts must be between 49.5 and 50.5 mm.*

Bolt E measures 49.4 mm, so it is not within the specified tolerance.

CHECK IT OUT!

3. A lacrosse ball must weigh 5.25 oz ± 0.25 oz. The weights of the lacrosse balls in one box are given in the table. Do all of the lacrosse balls weigh within the specified tolerance? If not, which lacrosse ball(s) are not within the specified tolerance? **no; C**

Ball	Weight (oz)
A	5.41
B	5.23
C	5.54
D	5.33
E	5.21

Tolerance can also be expressed as a percent. A measurement written as 5 mm ± 5% means that the measurement can be greater or less than 5 mm by an amount equal to 5% of 5 mm, or 0.25 mm. Therefore, the measurement can have a range of 4.75 mm–5.25 mm.

EXAMPLE 4 Using Tolerance Expressed as a Percent

Write the possible range of each measurement. Round to the nearest hundredth if necessary.

A **50 kg ± 2%**

$50(0.02) = 1$ *Find 2% of 50.*

50 kg ± 1 kg *Write the measurement and tolerance.*

49 kg–51 kg *Write the measurement as a range.*

B **125 lb ± 1.5%**

$125(0.015) = 1.875$ *Find 1.5% of 125.*

125 lb ± 1.88 lb *Write the measurement and tolerance. Round to the nearest hundredth.*

123.12 lb–126.88 lb *Write the measurement as a range.*

C **45 mm ± 0.3%**

$45(0.003) = 0.135$ *Find 0.3% of 45.*

45 mm ± 0.14 mm *Write the measurement and tolerance. Round to the nearest hundredth.*

44.86 mm–45.14 mm *Write the measurement as a range.*

CHECK IT OUT!

Write the possible range of each measurement. Round to the nearest hundredth if necessary.

4a. 4.1 in. ± 5% **4b.** 475 m ± 2.5% **4c.** 85 mg ± 0.5%

4a. 3.89 cm–4.31 cm
4b. 463.12 m–486.88 m
4c. 84.57 mg–85.43 mg

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Additional Examples

Example 3

Bright Days Blinds makes window shades. The width of a 30-inch shade should be within 0.18 in. of 30 in. A batch of shades has the widths shown in the table.

Shade	Width (in.)
A	30.06
B	29.75
C	29.84
D	30.12
E	29.93

Do all of the shades measure within the specified tolerance? If not, which shade(s) are not within the specified tolerance? No; B

Example 4

Write the possible range of each measurement. Round to the nearest hundredth if necessary.

A. 12 lb ± 3% 11.64 lb–12.36 lb

B. 15 oz ± 1.5% 14.77 oz–15.23 oz

C. 3 m ± 0.2% 2.99 m–3.01 m

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLES 3–4

- Why might tolerances be used in manufacturing?

3 Close

Summarize

Have students return to the dartboard diagrams from *Motivate*. Use the vocabulary they have learned to explain that player B is more precise because the darts consistently land close together, but less accurate because the darts are not near the bull's eye. Player A is less precise because the darts land further apart, but more accurate because the darts are clustered near the bull's eye.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. Precision has to do with how small the units of measure are. Accuracy has to do with how close the measured quantity is to the actual quantity.
2. Possible answer: the length of a handmade candlestick
3. See Additional Answers.

1-10 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**

- **Basic** 19–27, 37–51
- **Average** 19–27, 37–40, 43–48, 67
- **Advanced** 19–21, 27, 37–39, 43–47, 67

If you finished Examples **1–4**

- **Basic** 19–61, 64–66
- **Average** 19–40, 43–48, 52–68
- **Advanced** 19–21, 27–32, 37–39, 43–47, 52, 53, 60–69

Homework Quick Check

Quickly check key concepts.
Exercises: 20, 27, 28, 30, 38

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 10–12, 27–28, 60–61, 67–69

Construct viable arguments and critique the reasoning of others. Exercises 62–63

Attend to precision. Exercises 3–10, 19–27, 37–51, 66

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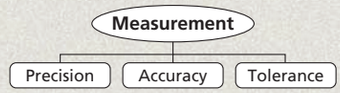
MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Explain the difference between precision and accuracy.
2. Describe a situation where the expected size of an object might be specified as 10 in. ± 0.5 in.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of when that characteristic of measurement would be important.



1-10 Exercises

GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. A ruler that can measure length to a smaller unit than another ruler is said to be more _____?_____. (*precise* or *accurate*) **precise**
2. A scale that gives a mass closer to the true mass of an object than another scale of the exact same type is said to be more _____?_____. (*precise* or *accurate*) **accurate**

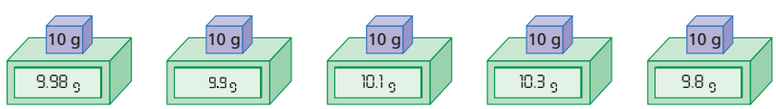
SEE EXAMPLE 1

Choose the more precise measurement in each pair.

3. 4 mL; 4.3 mL
4. 7 m; 6.8 m
5. 2.4 mg; 2.37 mg
6. 7 lb; 6.5 lb
7. 47 ft; 47.3 ft
8. 14 oz; 13.9 oz

SEE EXAMPLE 2

9. Sarah is comparing five different scales using a standard mass that is exactly 10 grams. Her results are shown below.



   a. Which scale is the most precise? **1**
   b. Which scale is the most accurate? **1**

10. A group of students compare the odometer readings on their bicycle computers after riding their bikes on a one-mile track. Their odometer readings are shown in the table. Whose odometer is the most precise? Whose is the most accurate? **Rasheed's; Jen's**

Student	Distance (mi)
Jen	1.01
Bill	0.97
Rasheed	0.989
Sasha	1.02

READING STRATEGIES

Precision and accuracy refer to two different characteristics of a measurement. Precision describes the level of detail in a measurement, whereas accuracy describes how close the measurement is to the true value.

Tolerance refers to the amount a measurement is allowed to vary from a specified value. For example, a specified value of 8 with a tolerance of ± 2 can be written 8 ± 2. Measurements within this tolerance are in the range 6–10. This range can be shown on a number line.

Write the range shown on each number line to the nearest tenth.

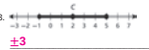
1. 8.0 ± 1.0

7.0–9.0

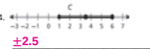
2. 3.2 ± 0.8

2.4–4.0

For 3 and 4, *C* represents the specified value. What is the tolerance?

3. 

±3

4. 

±2.5

For 5 and 6, a specified value is given. Which point is most accurate?

5. Specified value: 3.19

B

6. Specified value: 6.05

A

RETEACH

The *precision* of a measurement is determined by the smallest unit or fraction of a unit used.

Choose the more precise measurement in each pair.

a. 16.7 kg; 16.66 kg
Compare the two measurements: 16.7 kg is to the nearest tenth. 16.66 kg is to the nearest hundredth.
Because a hundredth of a kilogram is smaller than a tenth of a kilogram, 16.66 kg is more precise.

b. 8.5 km; 8532 m
Note that the units are different, but they can easily be converted:
8532 m = 8.532 km [1000 m = 1 km]
Compare the two measurements: 8.5 km is to the nearest tenth. 8.532 km is to the nearest thousandth.
Because a thousandth of a meter is smaller than a tenth of a meter, 8532 m is more precise.

Choose the more precise measurement in each pair.

1. 73.71 cm; 736.2 cm
   73.71 cm is to the nearest **hundredth** of a centimeter.
   736.2 cm is to the nearest **tenth** of a centimeter.
   73.71 cm is more precise.
2. 4732 mL; 4.73 L
   4732 mL = **4.732** L, which is to the nearest **thousandth** of a liter.
   4.73 L is to the nearest **hundredth** of a liter.
   4732 mL is more precise.
3. An object is weighed on three different scales. The results are shown in the table. Which scale is the most precise?

Scale	Measurement (lb)
1	44.9
2	45.105
3	45.01

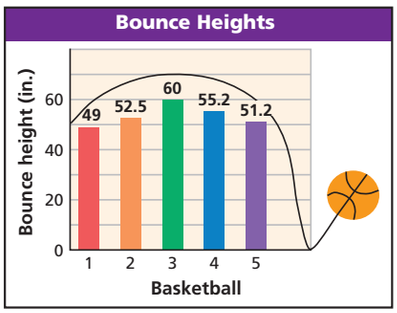
Scale 1 measures to the nearest **tenth** of a pound.
Scale 2 measures to the nearest **thousandth** of a pound.
Scale 3 measures to the nearest **hundredth** of a pound.
Scale **2** is the most precise.

SEE EXAMPLE 3

11. **Sports** A basketball for men's college games must have a mass of 595.5 ± 28.5 grams. Several basketballs are tested. Their masses are shown in the table. Do all of the basketballs fall within the specified tolerance? If not, which basketball(s) do not fall within the specified tolerance? **no; ball 4**

Basketball	1	2	3	4	5
Mass (g)	617.5	567.5	608	624.5	593.5

12. **Sports** A basketball for men's college games must bounce 51.5 ± 2.5 in. when dropped from a height of 6 feet. The bounce heights of several basketballs when dropped from a height of 6 feet are shown in the graph. Do all of the basketballs fall within the specified tolerance? If not, which basketball(s) do not have a bounce height within the specified tolerance? **no; balls 3 and 4**



SEE EXAMPLE 4

Write the possible range of each measurement. Round to the nearest hundredth if necessary.

13. 50 lb ± 2% **49 lb–51 lb**

14. 100 yd ± 0.5%

15. 25 cm ± 4%

16. 400 L ± 6% **376 L–424 L**

17. 250 mm ± 4% **240 mm–260 mm**

18. 70 kg ± 3% **67.9 kg–72.1 kg**

14. 99.5 yd–100.5 yd
15. 24 cm–26 cm

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
19–26	1
27	2
28	3
29–36	4

Choose the more precise measurement in each pair.

19. 4.33 g; 4337 mg **20.** 11 ft; 122 in. **21.** 6 tons; 11,000 lb **22.** 3 c; 2 pt

23. 67 mm; 6.83 cm **24.** 4.5 km; 3 mi **25.** 12 cm; 0.0127 m **26.** 7.23 lb; 115 oz

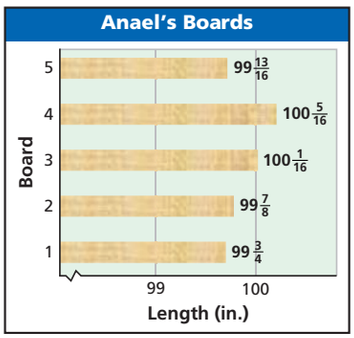
27. Maria is trying to beat the school record for the 400-meter dash. Her friends timed her using the stopwatch functions in their cell phones. The official track timer, which is highly accurate, reported that she ran the race in 51.12 seconds. Her friends recorded the times shown in the table.

Name	Time (s)
Lucy	51.1
Juan	52.23
Chandra	51.769
Pei	50.97

a. Who recorded the most precise time? **Chandra**

b. Who recorded the most accurate time? **Lucy**

28. Anael cut several boards to build a deck. The boards must be 100 in. ± 0.25 in. Her measurements of the boards after cutting them are shown in the graph. Which boards, if any, can she not use? **board 4**



PROBLEM SOLVING

Write the correct answer.

1. Rolondo is measuring the length of his lawn. Using a board that is 10 feet long, he measures his lawn to be 70 feet long. He then uses his foot, which is 12 inches long, to measure his lawn to be 864 inches. Which is the more precise measurement? Which is the more precise tool? **864 in.; Rolondo's foot**

2. A bolt used to assemble a car must have a length of 37.5 mm ± 4%. Does a bolt that is 39.3 mm long fall within the specified tolerance? Why or why not? **No; it is too long**

3. According to the Billiard Congress of America, BCA Equipment Specification, the diameter of a billiard ball is 2.25 inches with a tolerance of 0.005 inch. Which billiard ball(s) in the table below meet(s) this standard?

Ball	1	2	3	4	5
Diameter (in.)	2.255	2.249	2.251	2.250	2.2
Ball	6	7	8	9	10
Diameter (in.)	2.251	2.244	2.239	2.249	2.251
Ball	11	12	13	14	15
Diameter (in.)	2.250	2.219	2.247	2.257	2.288

1, 2, 3, 4, 5, 6, 9, 10, 11, 13

Select the best answer.

4. Ann is measuring the capacity of a 16-oz water bottle. She first uses a measuring cup and finds that the water bottle holds 16.2 oz of water. She then uses a graduated cylinder and finds that the water bottle holds 16.18 oz of water. Which is the more precise measurement? Which is the more precise tool? **D**
   A 16.2 oz; measuring cup
   B 16.2 oz; graduated cylinder
   C 16.18 oz; measuring cup
   D 16.18 oz; graduated cylinder

5. Ina added 32.155 milliliters (mL) of HCL to 64 mL of H_2O. How much solution does Ina have to the nearest milliliter? **G**
   F 95 mL H 97 mL
   G 96 mL J 98 mL

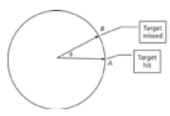
6. Jesse mixed 8.24 oz of paprika with 12.23 oz of pepper. How much of the spice combination does Jesse have to the nearest tenth of an ounce? **C**
   A 20.0 oz C 20.5 oz
   B 20.4 oz D 21.0 oz

7. An aquarium must be heated to 30˚C ± 1.5%. What is the acceptable temperature range for this aquarium? **H**
   F 25.5 ˚C–34.5 ˚C
   G 28.5 ˚C–31.5 ˚C
   H 29.55 ˚C–30.45 ˚C
   J 29.85 ˚C–30.15 ˚C

CHALLENGE

When aiming a projectile, such as an arrow, at a target, very small changes in aim can make a large difference in where the arrow lands. For example, the distance between the center and the edge of an archery target is about 2 ft. But if you aim your arrow at the center of the target and then move it two feet to one side, you will miss the target by a large distance!

The distance from an arrow's starting point to its landing point can be represented by the radius of a circle, with the starting point at the center of the circle and the landing point on the circle. In the diagram below, A represents the target and B represents where the arrow lands. The distance by which the arrow misses the target is the distance from A to B.



The circumference of a circle is $2\pi r$ times the radius. Therefore, the distance from A to B is $AB = \frac{\theta}{360} 2\pi r$.

1. William is 90 m from an archery target. His arrow lands 0.3 m left of the target. By how many degrees should he adjust his aim to hit the target? Round to the nearest hundredth of a degree. (*Hint:* $AB = 0.3$ m; $r = 90$ m) **0.19˚**

2. Sharon and Tony have built a catapult for the Medieval Festival. They enter a contest to hit a 20-ft target that is 250 ft away. How wide is the angle, in degrees, that the catapult can be aimed and hit the target? **4.6˚**

3. An engineer is launching a rocket to the moon. He accidentally enters the angle of the rocket's trajectory as 45.5˚, instead of 45.05˚. How far off target will the rocket be when it crosses the moon's orbit? Round to the nearest hundred kilometers. (*Hint:* The moon is about 384,000 km from Earth.) **3000 km**

PRACTICE B

Choose the more precise measurement in each pair.

1. 2.78 L; 2782 mL **2782 mL**
2. 6 ft; 72.3 in. **72.3 in.**
3. 2 c; 15 oz **15 oz**
4. 52 mm; 5.24 cm **5.24 cm**
5. 3 lb; 47 oz **47 oz**
6. 5.2 km; 5233 m **5233 m**

Write the possible range of each measurement. Round to the nearest hundredth if necessary.

7. 50 m ± 4% **48.00 m–52.00 m**
8. 90 °F ± 15% **76.50 °F–103.50 °F**
9. 15 L ± 2% **14.70 L–15.30 L**
10. 16 ft ± 1.5% **15.76 ft–16.24 ft**
11. 9 in. ± 10% **8.10 in.–9.90 in.**
12. 66 g ± 3% **64.02 g–67.98 g**

Use the following information for 13 and 14.

Marcel is measuring the volume of a liquid for chemistry class. He uses a beaker, a measuring cup, and a test tube. The teacher measures the liquid with a graduated cylinder, which gives the most accurate reading of 26.279 milliliters (mL). Marcel's measurements are shown below.

Measuring Device	Measurement (mL)
Beaker	26.3
Measuring Cup	25
Test Tube	26.21

13. Which device used by Marcel recorded the most precise measurement? **test tube**
14. Which device used by Marcel recorded the most accurate measurement? **beaker**

COMMON ERROR ALERT

In **Exercise 60,** students may find the differences between the given ranges and record the tolerance for length as ±1 in. and the tolerance for width as $\pm \frac{3}{4}$ in. Remind students that because tolerance is added and subtracted from the expected value, it will be *half* of the difference.

Teaching Tip **Visual** To think through **Exercise 63**, sketch the original board and the divisions marking the 8 sections. Suggest to students that the lines marking the divisions can represent the material that becomes sawdust while cutting the boards, and that those widths must be accounted for in the context of the real-life problem.

Answers

60. length: $5\frac{1}{2}$ in. $\pm \frac{1}{2}$ in.; width: $3\frac{7}{8}$ in. $\pm \frac{3}{8}$ in.

62. Possible answer: nearest $\frac{1}{2}$ inch; the manufacturer's meas. is to the nearest in., so the actual meas. is between $38\frac{1}{2}$ in. and $39\frac{1}{2}$ in. This means Linda's door must be at least $39\frac{1}{2}$ inches wide.

63. No; the precision is too great because (1) the length of the cut boards cannot be known to a greater precision than the length of the orig. board; (2) if Yusuf measures the cut boards the same way he measured the orig. board, the precision will be the same; and (3) part of the length of the orig. board equal to the thickness of the blade used to cut it will be lost.

Write the possible range of each measurement. Round to the nearest hundredth if necessary.

29. 45 lb ± 2% 44.1 lb–45.9 lb
30. 3 m ± 5% 2.85 m–3.15 m
31. 37 °C ± 1.5% 36.44 °C–37.56 °C
32. 750 kg ± 3% 727.5 kg–772.5 kg
33. 30 ft ± 4% 28.8 ft–31.2 ft
34. 550 mL ± 8% 506 mL–594 mL
35. 0.2 cm ± 5% 0.19 cm–0.21 cm
36. 0.25 kg ± 10% 0.23 kg–0.28 kg

Round each measurement to the specified precision.

37. 5456.3 mi to the nearest mile 5456 mi
38. 3.627 m to the nearest hundredth of a meter 3.63 m
39. 119.8 ft to the nearest ten feet 120 ft
40. 62.301 cg to the nearest tenth of a centigram 62.3 cg
41. 5,721 mg to the nearest kilogram 6 kg
42. 0.4586 km to the nearest meter 459 m

52. 100 m ± 2%
53. 50 g ± 4%

Choose the more precise measurement in each pair. If they are equally precise, write "neither."

43. 16.270 liters; 16,453.2 mL
44. 437 cm; 437 mm
45. 0.265 cm; 260 mm
46. 5.20 kg; 5200.0 mg
47. 55 yd; 165 ft
48. 67 min; 1.1 h
49. 33 mg; 0.033 g neither
50. 42.7 cm; 427.0 mm
51. 475.0 mL; 0.475 L

Rewrite each specified tolerance as a percent.

240 ft ± 5%
750 kg ± 2%

52. 100 m ± 2 m
53. 50 g ± 2 g
54. 240 ft ± 12 ft
55. 750 kg ± 15 kg
56. 25 in. ± 0.25 in. 25 in. ± 1%
57. 425 lb ± 8.5 lb 425 lb ± 2%
58. 60 oz ± 1.5 oz 60 oz ± 2.5%
59. 175 km ± 5.25 km 175 km ± 3%

LINK Technology

Automated equipment plays a large role in processing the approximately 584 million pieces of mail that the U.S. Postal Service delivers each day. Machines sort mail, cancel stamps, scan barcodes, and even "read" handwritten addresses.

Source: Postal Facts 2010, USPS

60. **Technology** Postcards that do not fit in the U.S. Postal Service's automatic sorting machines require additional postage for mailing. The machine will accept postcards whose length is between 5 and 6 inches and whose width is between $3\frac{1}{2}$ and $4\frac{1}{4}$ inches. Write these requirements as tolerances.

61. **Sports** For women's collegiate competition, a basketball's circumference, mass, and bounce height must fall within given tolerance levels of regulation measurements. The table shows these tolerance levels as well as measurements taken on five different basketballs. Which basketball meets all of the specified tolerances? ball 4

	Circumference (mm)	Mass (g)	Bounce Height (mm)
Tolerance	730.56 ± 6.5	538.5 ± 28.5	1358.5 ± 63.5
Basketball #1	729.8	509.3	1343.4
Basketball #2	723.5	529.8	1299.8
Basketball #3	734.2	542.6	1293.5
Basketball #4	725.5	528.0	1364.5
Basketball #5	740.0	555.9	1407.4

62. **Write About It** Linda wants to purchase a new sofa. Before buying the sofa, Linda must measure her doorway to make sure that the sofa will fit through the door. The sofa manufacturer says that the sofa measures 39 inches from front to back. What level of precision would you recommend Linda measure to? Explain.

63. **Critical Thinking** Yusuf measured a board and determined that it was 125.5 centimeters long. He then cut the board into eight equal pieces. His calculator shows that 125.5 ÷ 8 = 15.6875. Is it reasonable for Yusuf to record the length of the 8 smaller boards as 15.6875 centimeters? Explain why or why not.

©Steve Hamblin/Alamy

TEST PREP

64. The mass of a crystal is 0.9728 grams. What is the mass of the crystal to the nearest milligram?

(A) 1 milligram
(B) 9.73 milligrams
(C) 973 milligrams
(D) 972.8 milligrams

65. A piece used to assemble a computer must be 1.4 millimeters ± 0.02 millimeters in diameter. Which of the following measurements does NOT meet the specified tolerance?

(F) 1.420 millimeters
(G) 1.402 millimeters
(H) 1.382 millimeters
(J) 1.378 millimeters

66. Which measurement is most precise?

(A) 475.3 milliliters
(B) 475 milliliters
(C) 0.475 liter
(D) 0.5 liter

CHALLENGE AND EXTEND

Percent accuracy or *percent error* indicates how far a measurement is from the true value. An instrument that has 1.5% accuracy means that the measured value is within 1.5% of the true value.

67. A scale shows that a standard mass of exactly 5.000 grams has a mass of 5.002 grams. What is the percent accuracy of the scale? **0.04%**

68. A car odometer is accurate to within 0.5%. The odometer records the distance from Charlotte, North Carolina, to Orlando, Florida, as 525.3 miles. What is the range of possible values for the actual mileage? **522.7 mi–527.9 mi**

69. **Astronomy** A scientist measures the distance to the moon using a method that has a percent error of 0.02%. He finds that the distance at a particular time is 384,403 kilometers. What is the range of possible values for the actual distance? **384,326 km–384,480 km**

TEST PREP DOCTOR In **Exercise 64, B and D** can be eliminated because the numbers are not given to the nearest milligram. Students who selected **A** may have made an error converting units.

Journal

Have students explain a situation in which accuracy and precision are both important, and one in which other priorities might outweigh the cost of a high degree of accuracy and/or precision.

ALTERNATIVE ASSESSMENT

Have students use examples to describe the difference between precision and accuracy.

Power Presentations with PowerPoint®

1-10 Lesson Quiz

1. Choose the more precise measurement: 2.4 km; 2430 m 2430 m

2. Jorge works in a mail room. To test the accuracy of the scales in the mail room, he uses a weight that is *exactly* 8 oz. and gets the following results:

Scale 1: 8.02 oz
Scale 2: 7.8988 oz
Scale 3: 8.015 oz

a. Which scale is the most precise? 2

b. Which scale is the most accurate? 3

3. Monique is cutting wooden slats that are to be 20 in. long. Three slats have the following lengths:

A: 19.96875 in.
B: 19.875 in.
C: 20.0625 in.

a. Which slat(s), if any, do not fall within a tolerance of 0.0625 in.? B

b. Which slat(s), if any, do not fall within a tolerance of 0.5%? B

Also available online

SECTION 1B MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills a real-world format.

INTERACTIVE **Online Edition**

Answers

3. Maria; Paula took the additional $\frac{1}{4}$ off the original price, but it is actually off the reduced price. Since the reduced price is less than the original price, the additional discount will be less.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

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MATHEMATICAL PRACTICES **Make sense of problems and persevere in solving them.**

Percentages

Bargain Hunters Maria is on her high school's lacrosse team, and her friend Paula is on the softball team. The girls notice an advertisement in the newspaper for a clearance sale at their favorite sporting goods store. The ad shows an additional $\frac{1}{4}$ off the already reduced prices of 60% off. Maria and Paula head to the store to shop for bargains.

1. Maria finds a lacrosse stick with a regular price of $65. Find the sale price of the lacrosse stick prior to the additional $\frac{1}{4}$ off. **$26**
2. Find the sale price of Maria's lacrosse stick with the additional $\frac{1}{4}$ off. **$19.50**
3. Paula says that with the extra $\frac{1}{4}$ off, the total discount is 85% off. Maria thinks the discount is less than that. Who is correct? Explain your reasoning.
4. Paula finds a softball glove with a price tag that is not readable. The sales clerk scans the bar code and says the sale price, including the extra $\frac{1}{4}$ off, is $16.50. What was the original price of the softball glove? Show your reasoning. **$55**
5. Sales tax is 7.8%. Find the total amount that the girls will pay for the lacrosse stick and the softball glove together, including tax. **$38.81**

(cr),Sam Dudgeon/HMH; (bl),©COMSTOCK, Inc.; (br),©COMSTOCK, Inc.

INTERVENTION

Scaffolding Questions

1. What does $\frac{1}{4}$ off mean? The price has been reduced by $\frac{1}{4}$, or 25%.
2. How can this problem be solved using proportions? $\frac{3}{4} = \frac{x}{26}$
3. One of the girls is incorrect. What common error did she make? Paula added successive discounts instead of multiplying.
4. If the final sale price, including the extra 25% off, is $16.50, then $16.50 is what percent of the first sale price? 75
5. Explain at least two ways to calculate the price with tax included. Possible answer: Find the tax and add to the price, or multiply the price by 1.078.

Extension

What is the total discount at the sporting goods store, as a single percent? Explain how you found your answer. 70%; possible answer: If p represents the original price, then $0.4p$ is the price after the first discount of 60%. $0.75\ (0.4p) = 0.3p$ is the price after the additional 25% off. This is 70% less than the original price.

Quiz for Lessons 1-8 Through 1-10

1-8 Rates, Ratios, and Proportions

1. Last week, the ratio of laptops to desktops sold at a computer store was 2:3. Eighteen desktop models were sold. How many laptop models were sold? 12
2. Anita read 150 pages in 5 hours. What is her reading rate in pages per minute? $\frac{1}{2}$ page/min
3. Twenty-six crackers contain 156 Calories. Find the unit rate in Calories per cracker. 6 Calories/cracker
4. A store developed 1024 photographs in 8 hours. Find the unit rate in photographs per hour. 128 photographs/h

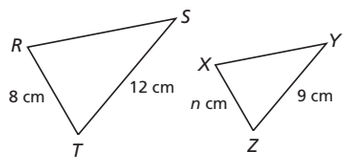
Solve each proportion.

5. $\frac{-18}{n} = \frac{9}{2}$ -4
6. $\frac{d}{5} = \frac{2}{4}$ 2.5
7. $\frac{4}{12} = \frac{r+2}{16}$ $\frac{10}{3}$
8. $\frac{-3}{7} = \frac{6}{x+6}$ -20

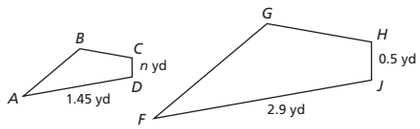
1-9 Applications of Proportions

Find the value of *n* in each diagram.

9. $\triangle RST \sim \triangle XYZ$ 6 cm



10. $ABCD \sim FGHJ$ 0.25 yd



1-10 Precision and Accuracy

Choose the more precise measurement in each pair.

11. 2.5 ft; 2 ft
12. 1 yd; 3 ft
13. 5910 g; 5.9 kg
14. 16 oz; 16.0 oz

Write the possible range of each measurement. Round to the nearest hundredth if necessary.

15. 300 m ± 1% 297 m–303 m
16. 150 lb ± 6% 141 lb–159 lb
17. 60 L ± 0.5% 59.7 L–60.3 L
18. 220 kg ± 1.5% 216.7 kg–223.3 kg

READY TO GO ON?

SECTION 1B

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources
Section Quiz

INTERVENTION

Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**

***Ready to Go On?* Online**

READY TO GO ON?

Diagnose and Prescribe

NO INTERVENE

YES ENRICH

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 1B: Worksheets	*Ready to Go On? Intervention*, Section 1B: Online	
Lesson 1-8	1-8 Intervention	Activity 1-8	
Lesson 1-9	1-9 Intervention	Activity 1-9	
Lesson 1-10	1-10 Intervention	Activity 1-10	

***Ready to Go On? Enrichment*, Section 1B**
Worksheets
Online

Organizer

Objective: Help students organize and review key concepts and skills in this chapter.

Online Edition
Multilingual Glossary

Resources

Learn It Online
Multilingual Glossary

Lesson Tutorial Videos

Answers

1. literal equation
2. ratio
3. constant
4. 1.99g
5. $t + 3$
6. 5
7. 5
8. 6
9. $150 \div m$; 30; 25; 15

CHAPTER 1 Study Guide: Review

Vocabulary

accuracy	formula	scale drawing
algebraic expression	identity	scale factor
constant	indirect measurement	scale model
conversion factor	literal equation	similar
corresponding angles	numerical expression	solution of an equation
corresponding sides	precision	tolerance
cross products	proportion	unit rate
dimensional analysis	rate	variable
equation	ratio	
evaluate	scale	

Complete the sentences below with vocabulary words from the list above.

1. A formula is a type of a(n) ___?___.
2. A(n) ___?___ is used to compare two quantities by division.
3. A(n) ___?___ is a value that does not change.

1-1 Variables and Expressions

EXAMPLES

■ **Barbara has saved d dollars for a $65 sweater. Write an expression for the amount of money she still needs to buy the sweater.**

$65 - d$ *Think: d dollars less than the price of the sweater.*

■ **Evaluate $b - a$ for $a = 7$ and $b = 15$.**

$b - a = 15 - 7$ *Substitute the values for the variables.*
$= 8$

EXERCISES

4. Grapes cost \$1.99 per pound. Write an expression for the cost of g pounds of grapes.
5. Today's temperature is 3 degrees warmer than yesterday's temperature t. Write an expression for today's temperature.

Evaluate each expression for $p = 5$ and $q = 1$.

6. qp 7. $p \div q$ 8. $q + p$

9. Each member of the art club will make the same number of posters to advertise their club. They will make 150 posters total. Write an expression for how many posters each member will make if there are m members. Find how many posters each member will make if there are 5, 6, and 10 members.

1-2 Solving Equations by Adding or Subtracting

EXAMPLES

Solve each equation. Check your answer.

■ $x - 12 = -8.3$

$+12 \quad +12$

$x = 3.7$

Check $x - 12 = -8.3$

$3.7 - 12$	-8.3
-8.3	-8.3 ✓

■ $-7.8 = 5 + t$

$-5 \quad -5$

$-12.8 = t$

Check $-7.8 = 5 + t$

-7.8	$5 + (-12.8)$
-7.8	-7.8 ✓

EXERCISES

Solve each equation. Check your answer.

10. $b - 16 = 20$

11. $4 + x = 2$

12. $9 + a = -12$

13. $-7 + y = 11$

14. $z - \frac{1}{4} = \frac{7}{8}$

15. $w + \frac{2}{3} = 3$

16. Robin needs 108 signatures for her petition. So far, she has 27. Write and solve an equation to determine how many more signatures she needs.

1-3 Solving Equations by Multiplying or Dividing

EXAMPLES

Solve each equation.

■ $\frac{z}{2.4} = 12$

$(2.4)\frac{z}{2.4} = (2.4)\,12$

$z = 28.8$

■ $-8x = 148$

$\frac{-8x}{-8} = \frac{148}{-8}$

$x = -18.5$

EXERCISES

Solve each equation. Check your answer.

17. $35 = 5x$

18. $-3n = 10$

19. $-30 = \frac{n}{3}$

20. $\frac{x}{-5} = -2.6$

21. $5y = 0$

22. $-4.6r = 9.2$

1-4 Solving Two-Step and Multi-Step Equations

EXAMPLE

■ **Solve** $\frac{3x}{5} - \frac{x}{4} + \frac{1}{2} = \frac{6}{5}$.

$\frac{3x}{5} - \frac{x}{4} + \frac{1}{2} = \frac{6}{5}$

$20\left(\frac{3x}{5} - \frac{x}{4} + \frac{1}{2}\right) = 20\left(\frac{6}{5}\right)$ *Multiply by the LCD.*

$12x - 5x + 10 = 24$

$7x + 10 = 24$ *Combine like terms.*

$-10 \quad -10$

$7x = 14$

$\frac{7x}{7} = \frac{14}{7}$

$x = 2$

EXERCISES

Solve each equation. Check your answer.

23. $4t - 13 = 57$

24. $5 - 2y = 15$

25. $\frac{k}{5} - 6 = 2$

26. $\frac{5}{6}f - \frac{3}{4}f + \frac{3}{4} = \frac{1}{2}$

27. $7x - 19x = 6$

28. $4 + 3a - 6 = 43$

29. If $8n + 22 = 70$, find the value of $3n$.

30. If $0 = 6n - 36$, find the value of $n - 5$.

31. The sum of the measures of two angles is 180°. One angle measures $3a$ and the other angle measures $2a - 25$. Find a. Then find the measure of each angle.

Answers

10. $b = 36$
11. $x = -2$
12. $a = -21$
13. $y = 18$
14. $z = \frac{9}{8}$
15. $w = \frac{7}{3}$
16. $27 + s = 108$; 81
17. $x = 7$
18. $n = -\frac{10}{3}$
19. $n = -90$
20. $x = 13$
21. $y = 0$
22. $r = -2$
23. $t = 17.5$
24. $y = -5$
25. $k = 40$
26. $f = -3$
27. $x = -\frac{1}{2}$
28. $a = 15$
29. 18
30. 1
31. $a = 41$; 123°; 57°

Study Guide: Review **85**

Answers

32. $x = -2$
33. $r = -2$
34. $a = 1$
35. $x = -\frac{2}{3}$
36. no solution
37. all real numbers
38. 9
39. $n = \frac{360}{C}$
40. $a = \frac{2S}{n} - \ell$
41. 3.7 gal
42. $x = 15, -27$
43. $y = 7, 3$
44. $y = 9, -9$
45. $x = 17.4, -6.6$
46. $g = -4, -8$
47. $x = \frac{5}{7}, -\frac{5}{7}$
48. $|x - 55| = 5$; min. speed: 50 mi/h; max. speed: 60 mi/h

1-5 Solving Equations with Variables on Both Sides

EXAMPLE

■ **Solve $x + 7 = 12 + 3x - 7x$.**

$x + 7 = 12 + 3x - 7x$

$x + 7 = 12 - 4x$ *Combine like terms.*

$+4x \quad +4x$

$5x + 7 = 12$

$-7 \quad -7$

$5x = 5$

$\frac{5x}{5} = \frac{5}{5}$

$x = 1$

EXERCISES

Solve each equation. Check your answer.

32. $4x + 2 = 3x$

33. $-3r - 8 = -5r - 12$

34. $-a - 3 + 7 = 3a$

35. $-(x - 4) = 2x + 6$

36. $\frac{2}{3}n = 4n - \frac{10}{3}n - \frac{1}{2}$

37. $0.2(7 + 2t) = 0.4t + 1.4$

38. One photo shop charges \$0.36 per print. Another photo shop charges \$2.52 plus \$0.08 per print. Juan finds that the cost of printing his photos is the same at either shop. How many photos does Juan have to print?

1-6 Solving for a Variable

EXAMPLE

■ **Solve $A = P + Prt$ for r.**

$A = P + Prt$

$-P \quad -P$

$A - P = Prt$

$\frac{A - P}{Pt} = \frac{Prt}{Pt}$

$\frac{A - P}{Pt} = r$

EXERCISES

Solve for the indicated variable.

39. $C = \frac{360}{n}$ for n

40. $S = \frac{n}{2}(a + \ell)$ for a

41. The formula $a = \frac{d}{g}$ gives the average gas mileage a of a vehicle that uses g gallons of gas to travel d miles. Use the formula to find how many gallons of gas a vehicle with an average gas mileage of 20.2 miles per gallon will use to travel 75 miles. Round your answer to the nearest tenth.

1-7 Solving Absolute-Value Equations

EXAMPLES

■ **Solve $3|y + 4| = 30$.**

$\frac{3|y + 4|}{3} = \frac{|30|}{3}$ *Divide both sides by 3.*

$|y + 4| = 10$

Case 1

$y + 4 = 10$

$-4 \quad -4$

$y = 6$

Case 2

$y + 4 = -10$

$-4 \quad -4$

$y = -14$

EXERCISES

Solve each equation. Check your answer.

42. $|x + 6| = 21$

43. $7|y - 5| = 14$

44. $3|y| + 4 = 31$

45. $12 = |x - 5.4|$

46. $|g + 6| + 12 = 14$

47. $|x| = \frac{5}{7}$

48. Jason is driving his car at 55 mi/h. He needs to keep his car within 5 mi/h of his current speed. Write and solve an absolute-value equation to find Jason's maximum and minimum speeds.

1-8 Rates, Ratios, and Proportions

■ **The ratio of skateboarders to bikers in an extreme sports contest is 7:2. There are 91 skateboarders. How many bikers are there?**

$\frac{\text{skateboarders}}{\text{bikers}} \rightarrow \frac{7}{2}$ *Write a proportion.*

$\frac{7}{2} \times \frac{91}{x}$ *Let x be the number of bikers.*

$7 \cdot x = 2 \cdot 91$ *Use cross products.*

$7x = 182$

$\frac{7x}{7} = \frac{182}{7}$ *Solve for x.*

$x = 26$

There are 26 bikers.

49. A recipe for a casserole calls for 2 cups of rice. The recipe makes 6 servings of casserole. How many cups of rice will you need to make 10 servings of casserole?

Use dimensional analysis to convert each rate. Round your answer to the nearest hundredth if necessary.

50. 30 cm/s to m/h

51. 75 ft/s to mi/min

Solve each proportion. Check your answer.

52. $\frac{n}{8} = \frac{2}{10}$

53. $\frac{2}{9} = \frac{12}{x}$

54. $\frac{3}{k} = \frac{9}{15}$

55. $\frac{1}{3} = \frac{x}{x-6}$

1-9 Applications of Proportions

■ **When Janelle stood next to the Washington Monument, she cast a 1.2-foot-long shadow, and the monument cast a 111-foot-long shadow. Janelle is 6 feet tall. How tall is the monument?**

$\frac{x}{111} = \frac{6}{1.2}$ *Write a proportion.*

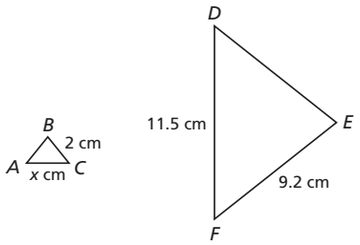
$1.2x = 666$ *Use cross products.*

$\frac{1.2x}{1.2} = \frac{666}{1.2}$ *Solve for x.*

The monument is 555 feet tall.

56. Find the value of x in the diagram.

$\triangle ABC \sim \triangle DEF$



57. A tree casts a shadow that is 14 ft long at the same time that a nearby 2-foot-tall pole casts a shadow that is 1.75 ft long. How tall is the tree?

58. A circle has a radius of 9 inches. The radius is multiplied by $\frac{2}{3}$ to form a second circle. How is the ratio of the areas related to the ratio of the radii?

1-10 Precision and Accuracy

EXAMPLE

■ **Choose the more precise measurement in the pair: 45.1 cm; 45.18 cm.**

45.1 cm *Nearest tenth of a centimeter*

45.18 cm *Nearest hundredth of a centimeter*

A hundredth of a centimeter is smaller than a tenth of a centimeter, so 45.18 cm is more precise.

EXERCISES

Choose the more precise measurement in each pair.

59. 1 ft; 12 in.

60. 37 g; 37.0 g

61. 550 cm; 5.5 km

62. 1.5 L; 1 L

Write the possible range of each measurement. Round to the nearest hundredth if necessary.

63. 500 lb ± 4%

64. 20 oz ± 0.5%

65. 75 kg ± 3%

66. 1035 mm ± 10%

Answers

49. $3\frac{1}{3}c$
50. 1080 m/h
51. 0.85 mi/min
52. $n = 1.6$
53. $x = 54$
54. $k = 5$
55. $x = -3$
56. 2.5
57. 16 ft
58. The ratio of the areas is the square of the ratio of the radii.
59. 12 in
60. 37.0 g
61. 550 cm
62. 1.5 L
63. 480 lb–520 lb
64. 19.9 oz–20.1 oz
65. 72.75 kg–72.25 kg
66. 931.5 mm–1138.5 mm

Study Guide: Review **87**

CHAPTER 1

CHAPTER TEST

Organizer

Objective: Assess students' mastery of concepts and skills in this chapter.

Online Edition

Resources

Assessment Resources

Chapter Tests

- Free Response (Levels A, B, C)
- Multiple Choice (Levels A, B, C)
- Performance Assessment

Learn It Online
State Resources Online

CHAPTER 1

CHAPTER TEST

Evaluate each expression for $a = 2$, $b = 3$, and $c = 6$.

1. $c - a$ **4**
2. ab **6**
3. $c \div a$ **3**
4. $\frac{c}{b}$ **2**
5. $b - a$ **1**
6. Give two ways write $n - 5$ in words. **five less than *n*; the difference of *n* and 5**
7. Nate runs 8 miles each week. Write an expression for the number of miles he runs in n weeks. Find the number of miles Nate runs in 5 weeks. **8*n*; 40 mi**

Solve each equation.

8. $y - 7 = 2$ **9**
9. $x + 12 = 19$ **7**
10. $-5 + z = 8$ **13**
11. $9x = 72$ **8**
12. $\frac{m}{-8} = -2.5$ **20**
13. $\frac{7}{8}a = 42$ **48**
14. $15 = 3 - 4x$ **−3**
15. $\frac{2a}{3} + \frac{1}{5} = \frac{7}{6}$ **1.45**
16. $8 - (b - 2) = 11$ **−1**
17. $-2x + 4 = 5 - 3x$ **1**
18. $3(q - 2) + 2 = 5q - 7 - 2q$ **no solution**
19. $5z = -3(z + 7)$ **−2.625**

Solve for the indicated variable.

20. $r - 2s = 14$ for s **$s = \frac{r - 14}{2}$**
21. $V = \frac{1}{3}bh$ for b **$b = \frac{3V}{h}$**
22. $P = 2(\ell + w)$ for ℓ **$\ell = \frac{P}{2} - w$**

Solve each equation.

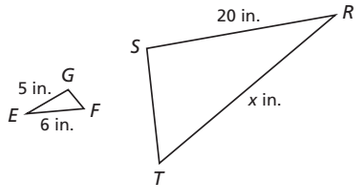
23. $|x - 14| = 21$ **35, −7**
24. $3|x| + 5 = 8$ **1, −1**
25. $|2v| = 6$ **3, −3**
26. Twenty-five students use 120 sheets of paper. Find the unit rate in sheets per student. **4.8 sheets/student**
27. Nutritionists recommend that teenagers consume 1300 milligrams of calcium per day. Use dimensional analysis to convert this rate to grams per year. **474.5 g/yr**

Solve each proportion.

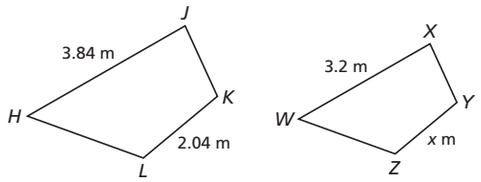
28. $\frac{5}{4} = \frac{x}{12}$ **15**
29. $\frac{8}{2z} = \frac{15}{60}$ **16**
30. $\frac{x + 10}{10} = \frac{18}{12}$ **5**
31. The scale on a map is 1 inch : 500 miles. If two cities are 875 miles apart, how far apart are they on the map? **1.75 in.**

Find the value of x in each diagram. Round your answer to the nearest tenth.

32. $\triangle EFG \sim \triangle RTS$ **24 in.**



33. $HJKL \sim WXYZ$ **1.7 m**



Choose the more precise measurement in each pair.

34. 6.5 oz; 6 oz
35. 16 oz; 1 lb
36. 3525 m; 3.5 km

Write the possible range of each measurement.

37. 25 ft ± 1% **24.75 ft – 25.25 ft**
38. 400 lb ± 4% **384 lb – 416 lb**
39. 250 cm ± 0.5% **248.75 cm – 251.25 cm**

College Entrance Exam Practice

CHAPTER 1

FOCUS ON SAT*

The SAT is often used to predict academic success at the college level. SAT scores are used to compare the math and verbal reasoning skills of students from all over the world.

You may want to time yourself as you take this practice test. It should take you about 8 minutes to complete.

HOT TIP! In each section of SAT questions, the easier questions are at the beginning of the section and harder questions come later. Answer as many of the easy questions as you can first, and then move on to the more challenging questions.

1. If $x - 3 = 4 - 2(x + 5)$, then $x =$?

(A) -3

(B) -1

(C) 1

(D) $\frac{3}{2}$

(E) $\frac{11}{3}$

2. A clothing store opens with 75 pairs of jeans on a sale table. By noon, 10 pairs have been sold. As of 2:00, another 8 pairs have been sold. A clerk then restocks with 12 pairs. Receipts show that 18 pairs of jeans were sold after 2:00. How many pairs of jeans are left at the end of the day?

(A) 23

(B) 27

(C) 36

(D) 51

(E) 123

3. Which of the following is equal to -3^4?

(A) 81

(B) 12

(C) -12

(D) -64

(E) -81

4. If Jack is three times as old as his sister Judy, which of the following expressions represents Jack's age if Judy is j years old?

(A) $3 \div j$

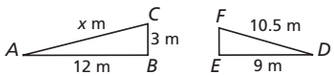
(B) $3j$

(C) $j + 3$

(D) $3 - j$

(E) $\frac{1}{3}j$

5. If $\triangle ABC \sim \triangle DEF$, what is the length of $\overline{AC}$?



(A) 2.6 meters

(B) 3.5 meters

(C) 7 meters

(D) 14 meters

(E) 15 meters

6. Which expression is equivalent to $8(6 + x)$?

(A) $48x$

(B) $8x + 14$

(C) $8x + 48$

(D) $x + 14$

(E) $x + 48$

*SAT is a registered trademark of the College Board, which was not involved in the production of, and does not endorse, this product.

College Entrance Exam Practice CHAPTER 1

Objective: Provide practice for college entrance exams such as the SAT.

Online Edition

Resources

College Entrance Exam Practice

Questions on the SAT represent the following content areas:

Number and Operation, 20–25%

Algebra and Functions, 35–40%

Geometry and Measurement, 25–30%

Data Analysis, Statistics, and Probability, 10–15%

Items on this page focus on:

- Number and Operation

TEST PREP DOCTOR

1. Students who chose **A** may have subtracted 3 from both sides instead of adding 3. Remind students that they should always check their answers to avoid errors caused by simple mistakes.

2. Students who chose **B** may have subtracted the 12 pairs restocked instead of adding them. Students who chose **C** may have found the total number of pairs of jeans sold for the day.

3. Students may choose **A** because they used -3 as the base of the power. Remind students that the exponent applies only to the number right before it, and that -3^4 can be thought of as $(-1)(3^4)$.

4. Students may choose **C** if they did not read the question carefully; Jack is three times as old as Judy, not three years older than Judy. Students who choose answer **E** may think that Judy is three times as old as Jack.

5. Students who chose **B** probably did not set up the proportion correctly. Remind students that in similar triangles, corresponding sides are proportional, and the similarity statement indicates which sides correspond.

6. Students may choose **E** because they do not understand the Distributive Property. Ask students about the meaning of the word *distribute* and discuss how it can be applied to an algebraic expression.

CHAPTER 1 TEST TACKLER

Organizer

Objective: Provide opportunities to learn and practice common test-taking strategies.

Online Edition

TEST PREP DOCTOR This Test Tackler focuses on using logic and estimation to eliminate answer choices. While this strategy may not always yield a single answer, it may save students time by eliminating some of the choices. In addition, it helps students analyze items before they begin to work.

Learn It Online
State Resources Online

Multiple Choice: Eliminate Answer Choices

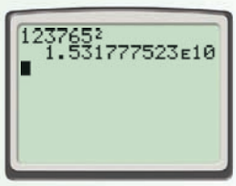
You can answer some problems without doing many calculations. Use logic to eliminate answer choices and save time.

EXAMPLE 1

Which number is the square of 123,765?

Ⓐ 15,317,775,225 Ⓒ 15,317,775,230

Ⓑ 15,317,775,233 Ⓓ 15,317,775,227



Your calculator will not help you on this question. Due to rounding, any of the answer choices are possible.

But you can use this fact to eliminate three of the answer choices:

The square of any number ending in 5 is also a number ending in 5.

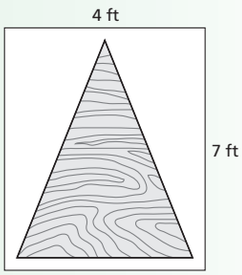
The only answer choice that ends in 5 is A, 15,317,775,225.

EXAMPLE 2

What is a possible area of the wooden triangle shown?

Ⓕ 11 square feet Ⓗ 14 square feet

Ⓖ 20 square feet Ⓙ 24 square feet



The triangle is inside a rectangle with an area of $7 \times 4 = 28$ square feet.

If the triangle had the same base and height as the rectangle, its area would be half the area of the rectangle, 14 square feet.

However, the triangle fits inside the rectangle, so its area must be less than 14 square feet.

The only answer choice that is less than 14 square feet is F, 11 square feet.

HOT TIP! Try to eliminate unreasonable answer choices. Some choices may be too great or too small, have incorrect units, or not be divisible by a necessary number.

Read each test item and answer the questions that follow.

Item A

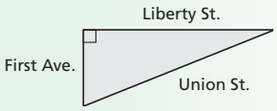
The top speed of a three-toed sloth is 0.12 miles per hour. About how many feet can a sloth travel in an hour?

Ⓐ 0.12 feet
Ⓒ 2.27 feet
Ⓑ 600 feet
Ⓓ 7500 inches

1. Are there any answer choices you can eliminate immediately? If so, which choices and why?
2. Describe how you can use estimation to find the correct answer.

Item B

A city park is shaped like a triangle. The Liberty Street side of the park is 120 feet long, and the First Avenue side is 50 feet long.



What is the approximate length of the side of the park that faces Union Street?

Ⓕ 25 feet
Ⓗ 65 feet
Ⓖ 110 inches
Ⓙ 130 feet

3. Can any of the answer choices be eliminated immediately? If so, which choices and why?
4. Are there any properties you can use to solve this problem? If so, what are they?
5. Describe how to find the correct answer without doing any calculations.

Item C

Approximately how long will the average 18-year-old have slept in his lifetime?

Ⓐ 6 weeks
Ⓒ 6 years
Ⓑ 6 months
Ⓓ 6 decades

6. Which answer choice can be eliminated immediately? Why?
7. Explain how to use mental math to solve this problem.

Item D

Sheila's paychecks for February and March were equal. If she worked every day during both months, for which month was her daily pay lower?

Ⓕ February
Ⓖ March
Ⓗ Her daily pay did not change.
Ⓙ Cannot be determined

8. What do you need to know to solve this problem?
9. Describe how you can find the correct answer.

Item E

Greg tripled the number of baseball cards he had last week. Which of these could be the number of cards Greg has now?

Ⓐ 100
Ⓒ 150
Ⓑ 200
Ⓓ 250

10. The number of cards that Greg has now must be divisible by what number? How can you tell if a number is divisible by this number?
11. Describe how to find the answer to this problem.

Answers to Test Items

A. B
B. J
C. C
D. G
E. C

Answers

Possible answers:

1. **D;** the units are incorrect
2. 0.12 mi is about $\frac{1}{10}$ mile, and 1 mile is about 5000 ft, so the answer should be close to $\frac{1}{10}(5000)$, or 500 ft. **B** is the only choice close to 500 ft.
3. **G;** the units are incorrect
4. Pythagorean Theorem, Triangle Inequality Theorem
5. Union Street is the hypotenuse of a right triangle, which means it is the longest side. So Union Street must be longer than 120 ft and 50 ft. **J** is the only choice greater than both of these distances.
6. **D;** 6 decades = 60 years, which is longer than 18 years.
7. People sleep 6–8 hours each day, or 6 to 8 out of each 24. This is about $\frac{1}{4}$ to $\frac{1}{3}$ of each day An 18-year-old will spend about $\frac{1}{4}$ to $\frac{1}{3}$ of 18 years asleep, which is about 4.5 to 6 years. **C** is the correct response.
8. the number of days in February and in March
9. February always has fewer days than March. If Sheila made the same amount of money each month, her daily pay must have been lower in March. **G** is the correct response.
10. 3; the sum of the digits will be divisible by 3.
11. Add the digits of each response and check if the sum is divisible by 3. **C** is the only choice divisible by 3.

Organizer

Objective: Provide review and practice for this chapter and standardized tests.

Online Edition

Resources

Assessment Resources

Chapter Cumulative Test

Learn It Online
State Test Practice

Learn It Online
State Resources Online

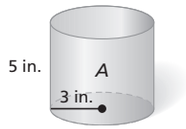
CUMULATIVE ASSESSMENT

Multiple Choice

1. What operation does ◊ represent if $x \lozenge 2.2 = 4.5$ when $x = 9.9$?

Ⓐ Addition
Ⓑ Subtraction
Ⓒ Multiplication
Ⓓ Division

2. Every dimension of cylinder *A* is multiplied by 4 to make cylinder *B*. What is the ratio of the volume of cylinder *A* to the volume of cylinder *B*?



Ⓕ $\frac{1}{64}$ Ⓗ $\frac{1}{4}$
Ⓖ $\frac{1}{16}$ Ⓙ $\frac{1}{3}$

3. A clock loses 5 minutes every day. How much time will it lose in 2 hours?

Ⓐ 0.417 second
Ⓑ 25 seconds
Ⓒ 240 seconds
Ⓓ 600 seconds

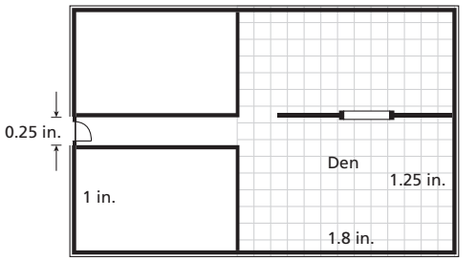
4. A statue is 8 feet tall. The display case for a model of the statue is 18 inches tall. Which scale allows for the tallest model of the statue that will fit in the display case?

Ⓕ 1 inch : 2 inches
Ⓖ 1 inch : 7 inches
Ⓗ 1 inch : 5 inches
Ⓙ 1 inch : 10 inches

5. What is the value of $-|6^2|$?

Ⓐ −36 Ⓒ −8
Ⓑ −12 Ⓓ −3

6. Mr. Phillips wants to install hardwood flooring in his den. The flooring costs $25.86 per square yard. The blueprint below shows his house. What other information do you need in order to find the total cost of the flooring?



Ⓕ The lengths and widths of the adjoining rooms in the blueprint
Ⓖ The total area of the blueprint
Ⓗ The scale of inches in the blueprint to yards in the house
Ⓙ The width of the den

7. What value of *n* makes the equation below have no solution?

$$2x + 2 = nx - 3$$

Ⓐ −2
Ⓑ 0
Ⓒ 2
Ⓓ 3

8. Which of the equations below represents the second step of the solution process?

Step 1: $3(5x - 2) + 27 = -24$
Step 2:
Step 3: $15x + 21 = -24$
Step 4: $15x = -45$
Step 5: $x = -3$

Ⓕ $3(5x + 27) - 2 = -24$
Ⓖ $3(5x + 25) = -24$
Ⓗ $15x - 2 + 27 = -24$
Ⓙ $15x - 6 + 27 = -24$

Answers

1. D
2. F
3. B
4. G
5. A
6. H
7. D
8. J
9. B
10. H
11. 6
12. 0.032
13. 64
14. 16.5
15. 3

If you are stuck on a problem, skip it and come back later. Another problem might remind you of something that will help. If you feel yourself become tense, take a few deep breaths to relax.

9. Cass drove 3 miles to school, and then she drove m miles to a friend's house. The total mileage for these two trips was 8 miles. Which equation CANNOT be used to determine the number of miles Cass drove?

Ⓐ $3 + m = 8$

Ⓑ $3 - m = 8$

Ⓒ $8 - 3 = m$

Ⓓ $8 - m = 3$

10. If $\frac{20}{x} = \frac{4}{x-5}$, which of the following is a true statement?

Ⓕ $x(x - 5) = 80$

Ⓖ $20x = 4(x - 5)$

Ⓗ $20(x - 5) = 4x$

Ⓙ $24 = 2x - 5$

Gridded Response

11. Four times a number is two less than six times the same number minus ten. What is the number?

12. Melissa invested her savings in a retirement account that pays simple interest. A portion of her account record is shown below. What is the interest rate on Melissa's account? Write your answer as a decimal.

Date	Transaction	Amount	Balance
8/1	Beginning deposit	$6000.00	$6000.00
8/31	Interest payment	$192.00	$6192.00
9/1	Withdrawal	$1000.00	$5192.00
9/30	Interest payment	$166.14	$5358.14

13. At 2:45 P.M. you are 112 miles from Dallas. You want to be in Dallas at 4:30 P.M. What is the average number of miles per hour you must travel to be on time?

14. A cyclist travels 45 miles in 4 hours. How many feet does she travel in one second?

15. A bike rental shop charges a one-time charge of \$8 plus an hourly fee to rent a bike. Dan paid \$24.50 to rent a bike for $5\frac{1}{2}$ hours. Find the bike shop's hourly fee in dollars.

Short Response

16. Alex buys 5 calendars to give as gifts. Each calendar has the same price. When the cashier rings up Alex's calendars, the total cost before tax is \$58.75.

a. Write and solve an equation to find the cost of each calendar.

b. The total cost of Alex's calendars after tax is \$63.45. Find the percent sales tax. Show your work and explain in words how you found your answer.

17. Fatima enrolled in a traveler rewards program. She begins with 10,000 bonus points. For every trip she takes, she collects 3000 bonus points.

a. Write a rule for the number of bonus points Fatima has after x trips.

b. Make a table showing the number of bonus points Fatima has after 0, 1, 2, 3, 4, and 5 trips.

c. When Fatima has collected 20,000 bonus points, she gets a free vacation. How many trips does Fatima need to take to get a free vacation?

Extended Response

18. Korena is putting a decorative border around her rectangular flower garden. The total perimeter of the garden is 200 feet.

a. Draw three different rectangles that could represent Korena's flower garden. Label the dimensions of your rectangles.

b. Use the table to show the lengths and widths of five different rectangles that could represent Korena's flower garden. Do not use any of your rectangles from part **a.**

Possible Dimensions of Korena's Garden		
Length (ℓ)	Width (w)	Perimeter (P)

c. The length of Korena's garden is 4 times its width. Explain how to use the perimeter formula $P = 2\ell + 2w$ to find the dimensions of Korena's garden.

d. Find the dimensions of Korena's garden.

Short-Response Rubric

Items 16–17

2 Points = The student's answer is an accurate and complete execution of the task or tasks.

1 Point = The student's answer contains attributes of an appropriate response but is flawed.

0 Points = The student's answer contains no attributes of an appropriate response.

Extended-Response Rubric

Item 18

4 Points = The student correctly draws three different rectangles in part **a,** lists five more different rectangles in part **b,** answers and explains part **c,** and answers part **d,** showing all work.

3 Points = The student correctly draws some rectangles in part **a** and lists some rectangles in part **b** but does not identify eight different possibilities, or the student correctly answers parts **a, b,** and **c** but does not correctly answer part **d.**

2 Points = The student answers all parts correctly but does not show any work or explanation, or the student correctly answers parts **a** and **b** but does not correctly answer parts **c** and **d.**

1 Point = The student finds four or fewer rectangles to fit the situation but does not correctly answer other parts of the problem, or the student attempts to answer all parts of the problem but does not correctly answer any part.

0 Points = The student does not answer correctly and does not attempt all parts of the problem.

Answers

16a. $5c = 58.75$; \$11.75

b. 8%

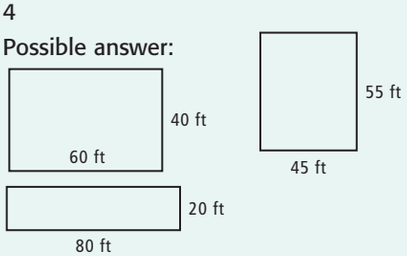
17a. $3000x + 10{,}000$

b.

Trips (x)	0	1	2	3	4	5
Points	10,000	13,000	16,000	19,000	22,000	25,000

c. 4

18a. Possible answer:



b. Possible answers:

Possible Dimensions of Korena's Garden		
Length (ℓ)	Width (w)	Perimeter (P)
30	70	200
25	75	200
50	50	200
15	85	200
10	90	200

c. Substitute $4w$ for ℓ in $P = 2\ell + 2w$ and solve for w. Substitute this value for w in $P = 2\ell + 2w$ and solve for ℓ.

d. 20 ft by 80 ft

Real-World CONNECTIONS

Organizer

Objective: Choose appropriate problem-solving strategies and use them to solve real-world problems.

Online Edition

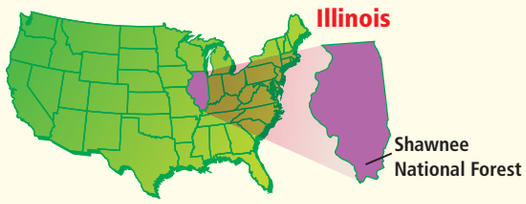
Bullfrogs

Reading Strategies

Remind students that information needed for a problem may not be in the question itself. **Problem 3** asks how far the largest bullfrog in Illinois could jump. Ask students where on the page it states the length of the largest bullfrog. in the introductory paragraph

Using Data Have students find how much farther a 5-inch bullfrog can jump than a 4-inch bullfrog. 15 in.

Learn It Online
State Resources Online



Bullfrogs

The bullfrog is the largest frog in Illinois. Adult bullfrogs are usually between $3\frac{1}{2}$ and 6 inches in length, but some have been known to grow even larger. The largest recorded bullfrog found in Illinois was 8 inches long and weighed more than a pound!

Choose one or more strategies to solve each problem.

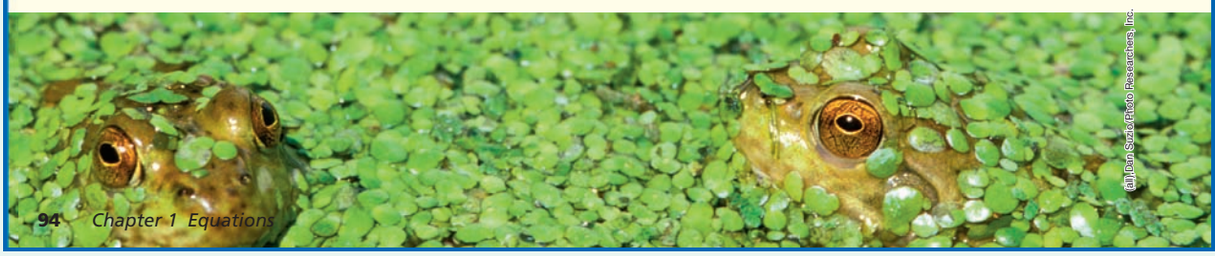
1. A bullfrog jumping up a hill is 8 feet from the top. Each time it jumps, it moves forward 18 inches. However, mud on the hill causes it to slide backward 2 inches after each jump. How many jumps will it take the frog to reach the top of the hill? **6**
2. A certain pond is home to 120 bullfrog tadpoles. This is 90 more than 3 times the number of adult bullfrogs that live in a second pond. How many adult bullfrogs live in the second pond? **10**

For 3–5, use the table.

3. How far could the largest recorded bullfrog in Illinois jump? **120 in.**
4. If a bullfrog is $4\frac{1}{2}$ inches long, what is its maximum jumping distance? **67.5 in.**
5. A certain adult bullfrog can jump $8\frac{3}{4}$ feet. This bullfrog must be at least how long? **7 in.**

Jumping Distances of Bullfrogs

Length of Bullfrog (in.)	Maximum Jumping Distance (in.)
4	60
5	75
6	90



(a) Tom Stack/Photo Researchers, Inc.

Problem-Solving Focus

For **Problem 1,** focus on the strategy "Use Logical Reasoning." Ask students how far the bullfrog actually advances after each jump. 16 in. Ask how many total inches he needs to travel. 96 in. Ask what operation will find the number of 16-inch jumps in 96 in. division

For **Problem 3,** students may apply the strategy "Find a Pattern" to the data in the table. They can use their pattern to answer the question. The numbers in the left column increase by 1. The numbers in the right column increase by 15.

The Great Snake Migration

When temperatures drop, large numbers of snakes leave the swamps of southern Illinois. Some travel just a few hundred feet, while others travel several miles, to Snake Road, a 3-mile-long road on the western edge of the Shawnee Forest. In this area, you can see as many as 40 to 50 snakes in a day, including cottonmouth water moccasins, rattlesnakes, and copperheads.

Choose one or more strategies to solve each problem.

1. In one hour, a researcher saw six snakes crossing Snake Road—three moccasins, a rattlesnake, a garter snake, and a diamondback water snake, though not in that order. The second snake to cross was a moccasin, and the other two moccasins crossed one after the other. The diamondback crossed before the garter snake. The first snake to cross was not the garter snake or the diamondback. A moccasin crossed last. In what order did the snakes cross Snake Road?
rattlesnake, moccasin, diamondback, garter, moccasin, moccasin

For 2, use the table.

In 2001–2002, several Illinois hospitals were asked how many vials of antivenom they had available to treat snake bites. The results are shown in the table. For example, 37 hospitals reported that they had between 1 and 4 vials of antivenom available.

Antivenom Survey Results						
Vials	0	1–4	5–9	10–14	15–19	20+
Hospitals		37	14	6	0	10

2. The number of hospitals with no antivenom available to treat snake bites was 6 less than 3 times the number of hospitals that had 1–4 vials. How many hospitals had no antivenom with which to treat snake bites? What percent of the total number of hospitals surveyed does this represent? Round your answer to the nearest percent. **105; 61%**

(cr) Victoria Smith/HMH; (tc) AP Photo/Charles Rex Arbogast; (br) Suzanne L. Collins/Photo Researchers, Inc.

The Great Snake Migration

Reading Strategies

ENGLISH LANGUAGE LEARNERS

Students may be unfamiliar with the word *antivenom.* Tell students that anti- means against. Ask students to use this information and context clues in the sentence to define both *venom* and *antivenom*. Venom is something in a snakebite that makes people sick; antivenom treats snakebites.

Using Data Discuss with students how to interpret the intervals in the table. Ask them what 20+ means. 20 or more

Problem-Solving Focus

Encourage students to use the four-step problem-solving process for **Problem 1.** Focus on the last step: **(4) Look Back.** After students think they have the correct order, have them read the problem again, placing a check mark next to each snake in their list as each sentence they read correctly describes its position.

Real-World Connections **95**